Adaptive Computer Testing For Online Courses

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Abstract **– Herewith we propose multi-step algorithm for computer-based testing allowing to differentiate and individualize academic performance rating procedure and subsequent testing path. Formulas to calculate quality points achieved by examinees are proposed. Exit criteria from the testing system are defined. A new function R_r was constructed to take decision based on three variables Y_{kn}, Y_e, Y_c specifying student's knowledge and skills.**

Index Terms **– Adaptability, testing, diagnostics, e-learning.**

I. Introduction

Tests as standard form of the tasks system with quantified results are widespread in various fields of our life. Advantages and disadvantages of the tests are obvious. One thing is quite clear: test is the most effective method of intellectual and personal qualities qualimetry, this is undoubted advantage of it. The novelty of present study is in formulas found to calculate the number of points achieved by examinees based on investigational studies.

II. TASK ASSIGNMENT

Bank of test items shall contain the entire volume of various tests of (light, medium, and advanced) complexity levels and different forms, allowing to define different levels of academic performance and individual students' learning paths. In turn, each level contains sublevel tasks a_i^j.

Present document proposed classification of level tests, consisting of 5 sub-levels. Simplified model to design tests level database using rating system of academic performance is proposed in present paper based on experimental data (Table 1.)

Table 1. Experimental data

Complexity levels	Response units	Sublevel tasks	Points	Digital equivalent	Mark
Third (High) complexity level	AND	a_1^3	95-100	4,0	Excellent
	OR	a_0^3	90-94	3,67	
Second (medium) complexity level	AND	a_2^2	85-89	3,33	Good
	OR	a_1^2	80-84	3,0	
	Unambiguous	a_0^2	75-79	2,67	
First (light) complexity level	Unambiguous	a_1^1	50-74	1.0-2,33	Satisfactory

Here for fixed level j sublevels $\boldsymbol{a_k^j}$, being the basic (primary) stage during testing, consist of the same sequence of educational elements $\boldsymbol{a_{ki}^j}(i = 1, \dots, m^j)$. Educational element here and further we understand the same concept or property, or definition, or both. Test questions relevant to educational elements $\boldsymbol{a_{ki}^j}(i = 1, \dots, m^j)$, are formed out of subset $\left\{\boldsymbol{a_{ki_m}^j}\right\}$ of randomized questions, where m is the number of questions relevant to the educational element i. It shall be noted that different response units and all three complexity levels may be included within the same test, but the subset $\left\{\boldsymbol{a_{ki_m}^j}\right\}$contains different questions on the educational elements of the course.

Level substantial database of the tests arranged in this manner allows to form level options for adaptive testing, individualize objective diagnosis path, construct decision-making function to assess student's academic performance and skills [1].

To check out minimum acceptable academic performance level tests shall contain light complexity questions $\boldsymbol{a^1}$. Such tasks are developed by the experts of relevant disciplines and shall be focused to check out quality of teaching as well as for the students who need to test their knowledge in particular subject or discipline and avoid them to fall down below the maximum permissible requirement level. (Fig. 1) While checking the completeness and depth of students' academic performance and skills in particular subject or discipline it is recommended to use tests of medium and advanced $\boldsymbol{a^2, a^3}$ complexity levels [2].

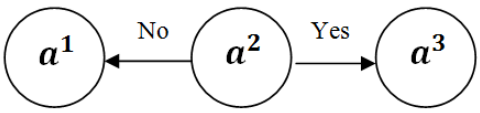


Fig. 1. Complexity levels

III. MODEL DEVELOPMENT

To improve assessment fairness of adaptive tests performance, multi-step algorithm of tasks selection and presentation shall be used when next step takes place only after preceding step results assessment. After performing current task by examinee each time it is necessary to take decision concerning complexity selection for the following tasks depending on correct or incorrect previous answer.

Theoretical model of adaptive academic performance

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control is based on students' academic performance level record depending on task complexity level. Algorithm of such record is composed in mathematical model of educational elements sequence construction, test items selection based on evaluated ranking of both individual tasks and entire sequences, deriving the final marks considering rated amount of points. (Fig. 2)

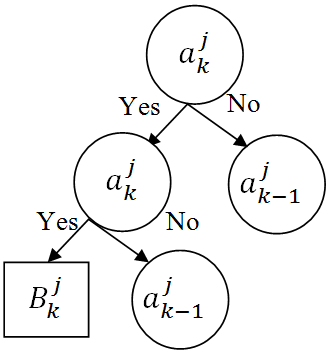


Fig 2.

Algorithm of tests' selection and presentation is based on feedback principle i.e. in case of correct answer the next task is selected more difficult, and wrong answer entails subsequent easier task comparing to the test with wrong answer. Multi-stage adaptive tests for each complexity level are designed upon branching strategy of adaptive testing. Proposed design includes: $j = 1,2,3;\ k = 2,1,0;\ a^j$- - tests of j^{th} complexity level; a_k^j- adaptive tasks of the k^{th} sub-level and j^{th} complexity level; $B_k^j -$ is the number of points achieved for tasks of k^{th} sub-level, j^{th} complexity level from the Bank of test tasks [1].

Various techniques of computer adaptive testing are available for the first test selection. The main rule for the first task selection is determined by the testing objective. In case of criterion-oriented testing the first task is usually selected close to the threshold of the highest complexity level, i.e. out of sublevel $\boldsymbol{a_2^2}$.

Now multistage testing algorithm will be described in more details, based on proposed graph of branching strategy of adaptive testing

From the database of sublevel $\boldsymbol{a_2^2}$ student gets two tasks $\bar{\boldsymbol{a}}_{\boldsymbol{2i}}^{\boldsymbol{2}}(i = 1, \dots, m^2)$, $\bar{\bar{\boldsymbol{a}}}_{\boldsymbol{2i}}^{\boldsymbol{2}}(i = 1, \dots, m^2)$ with identical m^2 sequence of educational elements $\boldsymbol{a_{2i}^2}(i = 1, \dots, m^1)$, composed of m^2 questions, whereas questions of test items relevant to training elements $\boldsymbol{a_{2i}^2}$, are formed from the subset $\{\boldsymbol{a_{2i_m}^2}\}$ by random selection.

Let assume that in both tasks presented $\bar{\boldsymbol{a}}_{\boldsymbol{2i}}^{\boldsymbol{2}}$ and $\bar{\bar{\boldsymbol{a}}}_{\boldsymbol{2i}}^{\boldsymbol{2}}$ student has given correct answer for m_2^2 questions with the same sequence number $\boldsymbol{k}_1, \dots, \boldsymbol{k}_{m_2^2}$. Therefore student gets

$$B_{cu}^2 = 89\frac{m_2^2}{m^2} \qquad (1)$$

points upon rating system, according to multiple selection on logic "AND" number of points is determined based on following formula

$$V_{22} = 2m_2^2 = \sum_{i=1}^{m^2} 2^{1-m_2^2} \prod_{j=1}^{5} c_{ij}, \qquad (2)$$

where $c_{ij} = 2$, if answer selected is correct; $c_{ij} = 0,$ if answer selected is wrong; $c_{ij} = 1$, if answer is not selected.

If $m^2 - m_2^2 = 0$ takes place then system recognizes good mark for measured academic performance and it brings transition to advanced level $\boldsymbol{a^3}$ of tests or student may exit the test system upon its own wish.

If $m^2 - m_2^2 > 0,$ where educational elements with atomic numbers $\boldsymbol{k}_1, \dots, \boldsymbol{k}_{m_2^1}$ are excluded than student is proposed with the test $\bar{\boldsymbol{a}}_{\boldsymbol{1i}}^{\boldsymbol{2}}(i = 1, \dots, m^2 - m_2^2)$, $\bar{\bar{\boldsymbol{a}}}_{\boldsymbol{1i}}^{\boldsymbol{2}}(i = 1, \dots, m^2 - m_2^2)$ with identical sequences of educational elements, composed of $m^2 - m_2^2$ questions formed by random selection out of $\{\boldsymbol{a_{1i_m}^2}\}$.

Let assume that student has given correct answer for both tasks $\bar{\boldsymbol{a}}_{\boldsymbol{1i}}^{\boldsymbol{2}}$ **and** $\bar{\bar{\boldsymbol{a}}}_{\boldsymbol{1i}}^{\boldsymbol{2}}$ in m_1^2 questions with identical serial numbers$\boldsymbol{p}_1, \dots, \boldsymbol{p}_{m_1^2}$. Therefore based on rating system student gets

$$84m_1^2 \qquad (3)$$

points, but if logic "OR" applied

$$min\{\bar{V}_{21}, \bar{\bar{V}}_{21}\} \qquad (4)$$

points, where $\bar{V}_{21} = \sum_{k=1}^{m^1 - m_2^1} \sum_{j=1}^{5} \bar{\sigma}_{2i}^{j}$, $\bar{\bar{V}}_{21} = \sum_{k=1}^{m^1 - m_2^1} \sum_{j=1}^{5} \bar{\bar{\sigma}}_{2i}^{j}$, $\bar{\sigma}_{2i}^{j}$- evaluation of answers for task $\bar{\boldsymbol{a}}_{\boldsymbol{1i}}^{\boldsymbol{2}}$, $\bar{\bar{\sigma}}_{2i}^{j} -$ evaluation of answers for task Therefore total number of points achieved is determined based on following formulas taking into account (1) -(4)

$$B_{cu}^1 = \frac{89m_2^2 + 84m_1^2}{m^2}, \quad V_{21} = 2m_2^2 + min\{\bar{V}_{21}, \bar{\bar{V}}_{21}\}. \qquad (5)$$

If $m^2 - m_2^2 - m_1^2 = 0$, then:

- with $85 \leq B_{cu}^1 \leq 89$ evaluation of measured academic performance system establishes as good, and allows to transfer to higher level $\boldsymbol{a^3}$ of test tasks, or student may exit from the test system upon his own wish.

- with $B_{cu}^1 < 85$ student gets B_{cu}^1, V_{21} points and test is exited.

If $m^2 - m_2^2 - m_1^2 > 0$ than educational elements of serial numbers $\boldsymbol{k}_1, \dots, \boldsymbol{k}_{m_2^2}, \boldsymbol{p}_1, \dots, \boldsymbol{p}_{m_1^2}$ are excludes from sublevel $\boldsymbol{a_0^2}$ and test $\bar{\boldsymbol{a}}_{\boldsymbol{0i}}^{\boldsymbol{2}}(i = 1, \dots, m^2 - m_2^2 - m_1^2)$, $\bar{\bar{\boldsymbol{a}}}_{\boldsymbol{0i}}^{\boldsymbol{2}}(i = 1, \dots, m^2 - m_2^2 - m_1^2)$ is proposed for the student with identical sequences of educational elements. Questions in these tasks are formed out of $\{\boldsymbol{a_{0i_m}^2}\}$ by random selection.

Let assume that student has given correct answer for each of presented tasks $\bar{\boldsymbol{a}}_{\boldsymbol{0i}}^{\boldsymbol{2}}$, $\bar{\bar{\boldsymbol{a}}}_{\boldsymbol{0i}}^{\boldsymbol{2}}$ for m_0^2 questions with matching serial numbers $\boldsymbol{q}_1, \dots, \boldsymbol{q}_{m_0^2}$. Therefore number of

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points achieved for tasks $\bar{\boldsymbol{a}}_{\boldsymbol{0i}}^{\boldsymbol{2}}$, $\bar{\bar{\boldsymbol{a}}}_{\boldsymbol{0i}}^{\boldsymbol{2}}$ is calculated based on following formula

$$79m_0^2\,, \tag{6}$$

and number of points with unambiguous selection is determined based on following formula

$$min\{\bar{V}_{20}, \bar{\bar{V}}_{20}\}. \tag{7}$$

Here $\bar{V}_{20}, \bar{\bar{V}}_{20}$ – number of points achieved for the tasks of $\bar{\boldsymbol{a}}_{\boldsymbol{0i}}^{\boldsymbol{2}}$, $\bar{\bar{\boldsymbol{a}}}_{\boldsymbol{0i}}^{\boldsymbol{2}}$:

$\bar{V}_{20} = \sum_{k=1}^{m^1 - m_2^2 - m_1^2} \sum_{j=1}^{5} \bar{\sigma}_{1i}^{j}$,

$\bar{\bar{V}}_{20} = \sum_{k=1}^{m^1 - m_2^1 - m_1^2} \sum_{j=1}^{5} \bar{\bar{\sigma}}_{1i}^{j}$,

where $\bar{\sigma}_{1i}^{j}$ is evaluation of answers for task $\bar{\boldsymbol{a}}_{\boldsymbol{0i}}^{\boldsymbol{2}}$, $\bar{\bar{\sigma}}_{1i}^{j}$ - evaluation of answers for task $\bar{\bar{\boldsymbol{a}}}_{\boldsymbol{0i}}^{\boldsymbol{2}}$. Therefore based on (5)-(7) total number of points is determined based on formulae:

$$B_{cu}^{0} = \frac{89m_2^2 + 84m_1^2 + 79m_0^2}{m^2}, \tag{8}$$

$$V_{20} = 2m_2^2 + min\{\bar{V}_{21}, \bar{\bar{V}}_{21}\} + min\{\bar{V}_{20}, \bar{\bar{V}}_{20}\}.$$

If $m^2 - m_2^2 - m_1^2 - m_0^2 = 0$, then:

- when $85 \le B_{cu}^{0} \le 89$ it is allowed to transfer to high level $\boldsymbol{a}^{\boldsymbol{3}}$ of test tasks or student may exit from the test system upon his own wish.
- when $B_{cu}^{0} < 85$ student gets B_{cu}^{0}, V_{20} points, and test system is exited.

If $m^2 - m_2^2 - m_1^2 - m_0^2 > 0$, then system proceeds to the easy level $\boldsymbol{a}^{\boldsymbol{1}}$. In this case, student is proposed with the tasks $\bar{\boldsymbol{a}}_{\boldsymbol{1i}}^{\boldsymbol{1}}(i = 1, \dots, m^2 - m_2^2 - m_1^2 - m_1^2)$ and $\bar{\bar{\boldsymbol{a}}}_{\boldsymbol{1i}}^{\boldsymbol{1}}(i = 1, \dots, m^1 - m_5^1 - m_4^1 - m_3^1)$ with identical sequences of training elements out of sublevel $\boldsymbol{a}_{\boldsymbol{1}}^{\boldsymbol{1}}$, where educational elements of serial numbers $\boldsymbol{k}_1, \dots, \boldsymbol{k}_{m_2^2}, \boldsymbol{p}_1, \dots, \boldsymbol{p}_{m_1^2}, \boldsymbol{q}_1, \dots, \boldsymbol{q}_{m_0^2}$ are excluded. Task questions $\bar{\boldsymbol{a}}_{\boldsymbol{1i}}^{\boldsymbol{1}}$ and $\bar{\bar{\boldsymbol{a}}}_{\boldsymbol{1i}}^{\boldsymbol{1}}$ are formed out of subset $\{\boldsymbol{a}_{\boldsymbol{1i_m}}^{\boldsymbol{1}}\}$ by random selection.

Let assume that student has given correct answer for questions $x_1, \dots, x_{m_2^1}$ for each proposed task $\bar{\boldsymbol{a}}_{\boldsymbol{1i}}^{\boldsymbol{1}}$, $\bar{\bar{\boldsymbol{a}}}_{\boldsymbol{1i}}^{\boldsymbol{1}}$. Herein number of points achieved for the tasks $\bar{\boldsymbol{a}}_{\boldsymbol{0i}}^{\boldsymbol{2}}$, $\bar{\bar{\boldsymbol{a}}}_{\boldsymbol{0i}}^{\boldsymbol{2}}$ is calculated based on formula

$$74m_1^1, \tag{9}$$

and number of points for tasks $\bar{\boldsymbol{a}}_{\boldsymbol{1i}}^{\boldsymbol{1}}$, $\bar{\bar{\boldsymbol{a}}}_{\boldsymbol{1i}}^{\boldsymbol{1}}$ with unambiguous selection is determined based on following formula

$$min\{\bar{V}_{11}, \bar{\bar{V}}_{11}\} = m_1^1. \tag{10}$$

Therefore based on (8)-(10) student gets the following points:

$$B_{clu}^{1} = \frac{89m_2^2 + 84m_1^2 + 79m_0^2 + 74m_1^1}{m^2}, \tag{11}$$

$$V_{11} = 2m_2^2 + min\{\bar{V}_{21}, \bar{\bar{V}}_{21}\} + min\{\bar{V}_{20}, \bar{\bar{V}}_{20}\} + min\{\bar{V}_{11}, \bar{\bar{V}}_{11}\}, \tag{12}$$

and test system is exited. Now let us define decision-making function depending on many independent variables that describes student's academic performance.

The first independent variable value chosen is degree of module (topic, discipline) questions digestion by the student. To determine the range of independent variable value from (2), (4), (7), (10) let us define number of points upon reactions blocks:

$$2m_2^2,\ min\{\bar{V}_{21}, \bar{\bar{V}}_{21}\},\ min\{\bar{V}_{20}, \bar{\bar{V}}_{20}\},\ min\{\bar{V}_{11}, \bar{\bar{V}}_{11}\}, \tag{13}$$

Due to the fact that maximum points for correct answers per each question are determined by the question evaluation, therefore maximum number of points achieved by students upon response units is defined based on following formulas

$$2m_2^2,\quad \sum_{k=1}^{m^2} \sum_{j=1}^{5} \sigma_{2i}^{j}, 2m_2^2,\ 2m_2^2, \tag{14}$$

where, σ_{2i}^{j} – correct answer evaluation rate. Therefore referring (13), (14), upon test results achieved by the student academic performance level in module (topic, discipline) can be defined by following formula:

$$Y_{kn} = \frac{m_2^2}{m^2} + \frac{min\{\bar{V}_{21}, \bar{\bar{V}}_{21}\}}{\sum_{k=1}^{m^2} \sum_{j=1}^{5} \sigma_{2i'}^{j}} + \frac{min\{\bar{V}_{20}, \bar{\bar{V}}_{20}\}}{2m^2} + \frac{min\{\bar{V}_{11}, \bar{\bar{V}}_{11}\}}{2m^2}, \tag{15}$$

where, $-1 \le Y_{kn} \le 1$.

The second independent variable value chosen is degree of questions digestion in module (topic, discipline). Let assume that out of m^2 questions within T_{exp} time student has given correct answer only for K_{ok} questions. Therefore based on test result degree of questions digestion in module (topic, discipline) can be defined based on following formula:

$$Y_{в} = \frac{K_{ok}}{m^2}, \tag{16}$$

where $0 \le Y_{в} \le 1$.

One of important criterion to assess student academic performance is degree of doubt in knowledge. Let assume C_c-level of doubt

Level of user doubt is calculated as an inverse ratio of conviction degree based on following formula 17.

$$C_c = U_u^{\alpha} \cdot (1 - S_S)^{\beta} \in [0,1],\ \alpha + \beta = 1 \tag{17}$$

The main principle of the method is to challenge decision comparing it with this type of decision. Defined latent parameter of this model uses the following quantitative parameters: subject, complexity, decision taken. Detailed

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description of user's doubt degree ratio calculation during computer-based testing is disclosed in papers [3], [4], [5], [6]. Now, using formulas (15) - (17) we can get decision-making function R_r, defining student's academic performance and skills in particular module.

$$R_r = \alpha Y_{kn} + \beta Y_в + \gamma Y_c, \quad (18)$$

where importance ratios $n \propto, \beta, \gamma$ are determined upon the linear system of algebraic equations referring to (11) in quantitative form.

III. CONCLUSION

Specified algorithm can be applied to the higher level of test items, and it is repeated while according to criterion specified assessment of academic performance is recognized by the student or system as satisfactory and test system is exited. In conditions of ubiquitous informational support and online study models implementation all above advantages of adaptive testing and online testing establish proposed model of academic performance control as the most acceptable and relevant.

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