Using LR-Type Membership Function For Estimating Fuzzy Linear Parameters For Electrical Load Estimation

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***Abstract*— a fuzzy linear estimation problem is formulated. The objective is to minimize the spread of the data points, taking into consideration the LR-type membership function of the fuzzy parameters to satisfy the constraints on each measurement point and to ensure that the original membership is included in the estimated membership. The proposed models are applied to different examples from the area of fuzzy linear regression and finally to different examples for estimating the electrical load on a busbar. It had been found that the proposed technique is more suited for electrical load estimation, since the nature of the load is characterized by the uncertainty and vagueness.**

Keywords*— *fuzzy regression, load estimation

I. Introduction

One of the most important requirements for active and secure operation of power distribution systems is the knowledge of the loads at different buses, especially the peak loads, which can be considered as the basis for power system state estimation and for technical and economical calculations. Accurate estimates for the peak loads result in accurate estimates of the system states, which yield to a secure operation for the load and the supply.

Reference [1] presents estimation for the electric load using fuzzy linear regression applied to the yearly peak load and the installed capacity of the consumers at the bus under study. The objective functions in this reference are the minimization of the spread of the data available. The linear programming based algorithm is the optimization tool used to solve the resulting minimization linear problem; in all the studied cases, a linear relation with fuzzy coefficients is assumed.

In this paper we present a new formulation for the fuzzy linear estimation problem. The problem is formulated as a linear programming problem, where the objective is to minimize the spread of the data points, taking into consideration the type of the membership of the fuzzy parameters, subject to satisfying constraints on each measurement point to insure that the original membership is included in the estimated membership. Different models are developed for a fuzzy triangular membership as well as the fuzzy numbers of LR-type. The proposed models are applied to different examples from the area of fuzzy linear regression and finally to different examples for estimating the electrical load on a substation busbar. It has been found that the proposed technique is more suited for the electrical load estimation, since the nature of the load is characterized by the uncertainty and vagueness.

II. Fuzzy Linear Regression

Fuzzy parameters linear estimation model or fuzzy regression model can be described by the following equation:

$$Y = f(x, \underline{A}) = \underline{A_1} x_1 + \underline{A_2} x_2 + \cdots + \underline{A_n} x_n \quad (1)$$

At any observation j; j = 1,2..., m, equation (1) can be written as

$$Y_j = f(x, \underline{A}) = \underline{A_1} x_{1j} + \underline{A_2} x_{2j} + \cdots + \underline{A_n} x_{nj} \quad (2)$$

In fuzzy regression, the differences between the observed and the estimated values are assumed to be due to the inherent ambiguity in the system. Therefore, the above fuzzy regression model is built in terms of possibilities. It evaluates all observed values as possibilities the system must contain. The model in equation (1) is named as a possibilstic regression model. In this model Y_j is the observation at measurement$j$. This output observation may be a non-fuzzy or a fuzzy observation. $A_i, i = 1,2,\dots,n$ are the fuzzy coefficients of the model in the form of LR-type (Trapezoidal) as (p_i, c_i^L, c_i^R) and x_{ij} is the input to the model where $i = 1,\dots,n$ and $j = 1,2,\dots,m$. In this section, two cases for the output Y_j are studied:

A. *Non-fuzzy output ($Y_j = m_j$) [5, 6]*

In this model the output Y_j is a non-fuzzy observation, but the model coefficients $A_i, i = 1,2,\dots,n$ are fuzzy parameters in the form of $A_i = (p_i, c_i^L, c_i^R), i = 1,\dots,n$ for the LR-type and the input x_{ij} is a non-fuzzy input. The membership function of A_j of the LR-Type is assumed to be trapezoidal function as shown in Figure 1. In general the membership function for the LR-type can be described as

$$\mu_{A_j} = f(x) = \begin{cases} L\left(\frac{p_j - x}{c_j^L}\right), & for\ x \le p_j \\ R\left(\frac{x - p_j}{c_j^R}\right), & for\ x \ge p_j \end{cases} \quad (3)$$

p_j is called the middle of A_j or the mean, c_j^L is the left spread and c_j^R is the right spread. Equation (1) can now be written as:

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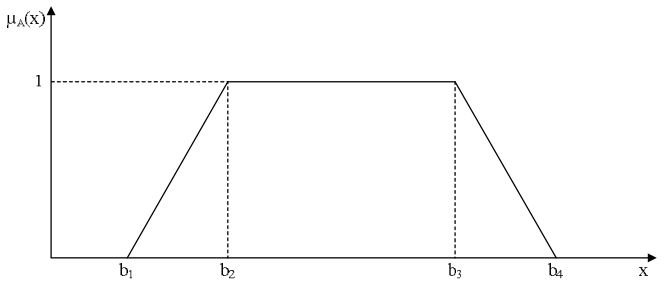


Fig 1. Trapezoidal Membership Function of A_j

$$Y_j = (p_1, c_1^L, c_1^R)x_{1j} + \cdots + (p_n, c_n^L, c_n^R)x_{nj}, j = 1, \ldots, m \quad (4)$$

for the (Trapezoidal) type of the fuzzy coefficients.

In the non-fuzzy output data regression described by equations (6), the parameters are to be found $A_i = (p_i, c_1^L, c_1^R)$ that minimize the spread of the fuzzy output for all data sets. For the fuzzy coefficients of the LR-type, the cost function to be minimized is:

$$J_1 = \sum_{j=1}^{m}\sum_{j=1}^{n}\left|2m_j - 2p_j x_{ij} + c_i^L x_{ij} - c_i^R x_{ij}\right| \quad (5)$$

Subject to satisfying the following two constraints on each data point:

$$Y_j \geq \sum_{i=1}^{n} p_i x_{ij} - (1-\lambda)\sum_{i=1}^{n} c_i^L x_{ij}, j = 1, \ldots, m \quad (6)$$

$$Y_j \leq \sum_{i=1}^{n} p_i x_{ij} + (1-\lambda)\sum_{i=1}^{n} c_i^R x_{ij}, j = 1, \ldots, m \quad (7)$$

Equations (5) to (7) formulate a linear optimization problem, which can be solved by linear programming using the simplex method. However, if the sum of the absolute value deviations in equation (5) is to be minimized, subject to satisfying the inequality constraints given by equations (6) and (7), then the problem is a least absolute value linear optimization and can be solved by using the software package available in the IMSL/STAT library.

B. Fuzzy Output [4,5]

If the output is a fuzzy number, it may be represented by a fuzzy number in the form of $Y_j = (m_j, \alpha_j^L, \alpha_j^R), j = 1, \ldots, m$, in the case of trapezoidal membership function. For LR-type (Trapazoidal), equation (2) can be written as:

$$(m_j, \alpha_j^L, \beta_j^R) = (\sum_{i=1}^{n} p_i x_{ij}, \sum_{i=1}^{n} c_i^L x_{ij}, \sum_{i=1}^{n} c_i^R x_{ij} \quad (8)$$

Equation (21) can be separated into the following equations:

$$m_j = \sum_{i=1}^{n} p_i x_{ij}, j = 1, \ldots, m \quad (9)$$

$$\alpha_j^L = \sum_{i=1}^{n} c_i^L x_{ij}, j = 1, \ldots, m \quad (10)$$

$$\beta_j^R = \sum_{i=1}^{n} c_i^R x_{ij}, j = 1, \ldots, m \quad (11)$$

The objective function to be minimized is given [81] as:

$$J_1 = 0.25\sum_{i=1}^{m}\left|4m_j - 4\sum_{i=1}^{n} p_i x_{ij} - \alpha_j^L + \sum_{i=1}^{n} c_i^L x_{ij} - \beta_j^R - \sum_{i=1}^{n} c_i^R x_{ij}\right| \quad (12)$$

subject to satisfying the following constraints:

$$m_j - (1-\lambda)c_j^L \geq \sum_{i=1}^{n} p_i x_{ij} - \sum_{i=1}^{n} c_i^L x_{ij}, j = 1, \ldots, m \quad (13)$$

and

$$m_j + (1-\lambda)c_j^R \geq \sum_{i=1}^{n} p_i x_{ij} - \sum_{i=1}^{n} c_i^R x_{ij}, j = 1, \ldots, m \quad (14)$$

Again the problem formulated in equations (12) to (14) for LR-type all are linear optimization problems subjected to a set of linear constraints. These problems can be solved using the standard linear programming using the simplex method. However, if the objective functions are the minimization of the sum of the absolute value of the deviation, then the least absolute value optimization technique based on linear programming can be used to solve the problems formulated above. The following examples illustrate the application of the different models, for the fuzzy parameter estimation, developed in the previous sections.

C. Example 1

This example is solved in reference [9] for the LR-type; the solution obtained was based on minimum least absolute deviation. The data for the TFN fuzzy output are listed in Table 1. The model required to fit these data points is in the form of:

$$\underline{Y_j} = \underline{A_0} + \underline{A_1}x_j + \underline{A_1}x_j^2, j = 1, \ldots, 16$$

The cost function to be minimized in this case is given as

$$J = 0.25\sum_{j=1}^{16}(4m_j - \alpha_j^L + \beta_j^R) - 0.25(4p_0 + 96p_1 + 148.4p_2 - c_0^L - 24c_1^L - 96.6c_2^L + c_0^R + 24c_1^R + 96.6c_2^R) \quad (15)$$

The cost function to be minimized is:

$$J = 0.25[(4p_0 + 96p_1 + 148.4p_2 - c_0^L - 24c_1^L - 96.6c_2^L + c_0^R + 24c_1^R + 96.6c_2^R)]$$

subject to satisfying the inequality constraints given as:

TABLE I. DATA FOR EXAMPLE 1

No.	x_j	$Y_j=(m_j, \alpha_j^L, \beta_j^R)$	No.	x_j	$Y_j=(m_j, \alpha_j^L, \beta_j^R)$
1	0.0	(11.5, 3., 2.5)	9	1.6	(84., 15., 16.)
2	0.2	(24.8, 4.5, 4.)	10	1.8	(82., 15., 16.)
3	0.4	(40., 6., 7.)	11	2.0	(103.7, 16., 17.)
4	0.6	(45.2, 7., 7.)	12	2.2	(102.6, 16., 17.)
5	0.8	(49.1, 9., 9.)	13	2.4	(103.1, 16., 17.)
6	1.0	(70., 11., 12.)	14	2.6	(111., 17., 19.)
7	1.2	(70.9, 12., 12.)	15	2.8	(109., 17., 19.)
8	1.4	(80.1, 14., 15.)	16	3.0	(121.7, 18., 21.)

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$$m_j - (1-\lambda)c_j^L \geq (p_0 + p_1 x_{1j} + p_2 x_{2j}^2) - (c_0^L + c_1^L x_{1j} + c2Lx2j2, j=1,\ldots,16 \quad (16)$$

$$m_j + (1-\lambda)c_j^L \leq (p_0 + p_1 x_{1j} + p_2 x_{2j}^2) + (c_0^L + c_1^L x_{1j} + c2Lx2j2, j=1,\ldots,16 \quad (17)$$

Note that, the number of parameters to be estimated is 9 and the number of inequality constraints is 32. The solution to the fuzzy parameters for the proposed model have been found to be $\underline{A_0} = (12.75, 2.75, 0.0)$, $\underline{A_1} = (42.1, 0, 0)$ and $\underline{A_2} = (140.794, 144.3, 0.0)$. With an alarm from the linear program, this solution is not the unique solution. The model equation in this case can be written as:

$$\underline{Y} = (12.75, 2.75, 0) + (42.1, 0, 0) + (140.794, 144.3, 0)x^2$$

This model satisfies all the constraints on the fuzzy optimization problem formulated above**.**

D. Example 2

The fuzzy linear parameter estimation algorithm, proposed in the previous sections, is implemented for determination of the required distribution system under uncertain conditions. The uncertainty appears at input, at output, and in the nature of the system itself. The following measured data is given in Table 2, where E_r is the yearly energy consumption, P_i is the installed capacity of electrical equipment at customers and P_r is the yearly peak load. [1]. The yearly peak load is given as a LR-type in the table below, Table 3. It is required to model this load as:

$$P_s = \underline{A_0} + \underline{A_r} E_r$$

where $\underline{A_0} = (p_0, c_0^L, c_0^R)$ and $\underline{A_1} = (p_1, c_1^L, c_1^R)$. Using the cost function defined in equation (15) and the constraints defined in equations (16) and (17), for this linear model, we obtain the following results:

$$P_s = (0.0, 0.0, 0.0) + (2.134, 1.974, 0.0)E_r$$

III. CONCLUSION

Different formulations for the fuzzy parameter estimation problem are presented. Different objective functions and constraints are derived for the membership functions. Two cases are studied, the first case when the output is a nonfuzzy data, the second case when the output is a fuzzy data following LR-type fuzzy output. Different examples from the area of the fuzzy regression analysis are offered in the text. Finally the proposed models are implemented for estimating the electrical load in a substation. It has been shown that due to uncertainty of the nature of the load, the proposed algorithm is well suited to estimate the electrical load in the power systems.

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TABLE II. POWER MEASURED IN SUBSTATION

#	E_r (MWh)	P_i (kW)	P_r (kW)
1	21.79	125	30.0
2	60.20	247	27.9
3	60.72	436	40.5
4	65.01	406	39.6
5	70.00	265	42.0
6	70.55	251	29.2
7	72.30	520	42.0
8	79.05	540	42.0
9	80.39	310	30.9
10	114.00	443	57.0
11	114.45	573	57.5
12	125.00	438	37.2
13	148.00	578	55.5
14	162.10	610	59.6

TABLE III. SAMPLES OF MEASUREMENTS IN S.S

#	E_r (MWh)	P_i (kW)	P_r (kW)
1	21.79	125	(30.0, 25.0, 33.0)
2	60.20	247	(27.9, 24.0, 30.5)
3	60.72	436	(40.5, 34.8, 44.9)
4	65.01	406	(39.6, 36.1, 43.0)
5	70.00	265	(42.0, 38.0, 45.7)
6	70.55	251	(29.2, 26.0, 33.0)
7	72.30	520	(42.0, 37.6, 45.0)
8	79.05	540	(42.0, 38.5, 46.0)
9	80.39	310	(30.9, 27.0 ,34.5)
10	114.00	443	(57.0, 52.5, 60.9)
11	114.45	573	(57.5, 52.8, 61.4)
12	125.00	438	(37.2, 34.4, 40.8)
13	148.00	578	(55.5, 52.0, 59.8)
14	162.10	610	(59.6, 55.3, 64.7)

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