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State Space Model of an Aircraft Using Simulink

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Abstract: **In aircraft designing it is crucial to understand and evaluate the behavior, performance, safety and other aspects of the systems before its operational testing. To develop a robust control system of an aircraft it is therefore necessary to have a good mathematical model of the system so that the behavior can be easily understood, predicted and controlled. In this paper the state space model of an autopilot aircraft is formulated using Simulink Matlab Function files. The proposed simulation model can be used to gain knowledge in order to make decisions at all development stages. In this paper the model has some assumptions such as the aircraft is taken in steady cruise at constant altitude and velocity, balancing the thrust, drag, weight and lift forces in *X* and *Y* direction, secondly change in pitch angle will not change the speed of the aircraft under any circumstances and lastly he input will be the elevation differential θ_d and the output will be the pitch angle θ_p of the aircraft. We have worked only on stability axis and emphasized on the fast settling of the system without significant overshoots. The state space model using Simulink is successfully formulated and simulated with and without controller. The angle of aircraft, pitch rate and pitch angle are selected as state variables.In addition to this the stability analysis from the state space model is also performed and improved using PD controller.**

I. INTRODUCTION

Models are usually made to organize disparate information into coherent whole, to think and calculate logically about which components and interactions are important in a complex system, to discover new strategies, to make important corrections to the conventional wisdom and to understand the essential qualitative features [1]. Models basically are either a graphical, mathematical (Symbolic), physical or verbal representation or simplified version of a concept, phenomenon or structure of a system. It can facilitate understanding of the overall system by eliminating unnecessary components. It can aid in decision making providing different scenarios. It can help to explain, control and predict events on the basis of past behavior and observation of the system[3].

Today's aircraft designs rely heavily on automatic control system to monitor and control many of aircraft's subsystem. The development of automatic control system has played an important role in the growth of civil and military aviation. For this situation an autopilot is designed that controls the pitch of aircraft that can be used by the flight crew to lessen their workload during cruising and help them land their aircraft during adverse weather condition in the real situation. Aircraft development is crucial to understand and evaluate behavior, performance, safety and other aspects of the systems before and after they are physically available for testing. Simulation models are used to gain knowledge in order to make decisions at all development stages [10].

An aircraft can be modeled by a set of very complicated six non linear coupled differential equations [3]. Observing the air craft model given below in Figure 1.1, the following state variables are obtained:

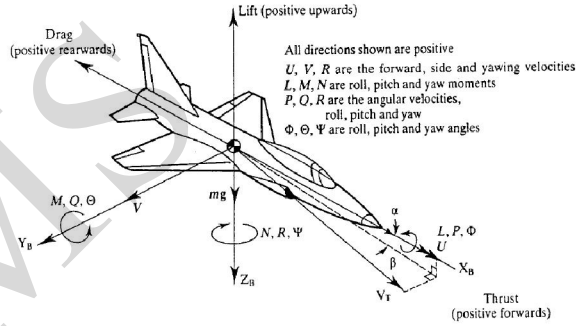


Fig 1. Aircraft Variables

$$\text{aircraft velocity vector } V=\begin{bmatrix} U \\ V \\ W \\ P \\ Q \\ R \end{bmatrix}$$

Where,

U is the longitudinal forward velocity
V is the lateral velocity
W is the vertical velocity
P is the roll rate
Q is the pitch rate
R is the Yaw rate

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The forces and moments are defined in a similar manner,

$$F=\begin{bmatrix} X \\ Y \\ Z \\ L \\ M \\ N \end{bmatrix}$$ Where,

X is the longitudinal force
Y is the transverse force
Z is the vertical force
L is the roll moment
M is the pitch moment
N is the yaw moment

An aircraft has six degree of freedom which means it has six paths free to follow: it can move forward, sideways, down, it can rotate about its axes with yaw, pitch and roll. In order to describe the state of a system that has six degree of freedom, values of six variables are necessary [10]. However under certain assumption, the equation of aircraft motion can be decoupled and linearized into longitudinal and lateral equation [14]. In this research paper an auto pilot carrying the pitch of the aircraft in analyzed and the state space is formulated. The assumptions made in this study are as follows:

i. Aircraft is in steady cruise at constant altitude and velocity, balancing the thrust, drag, weight and lift forces in X and Y direction [11].
ii. A change in pitch angle will not change the speed of the aircraft under any circumstances [13].
iii. The input will be the elevation differential θ_d and the output will be the pitch angle θ_p of the aircraft [11].

A model is a simplified representation of a real-world system; a simulation is an execution of a model using input data to extract information from the model. The simulation can be a real-life or hypothetical situation. By analogy with an experiment in the real world on a system that needs atmospheric conditions and delivers measurement data, the model also needs inputs and delivers simulation data. The simulation enables the prediction of behavior of the system from a set of parameters and initial conditions [15]. There are many reasons to simulate instead of setting up an experiment on a system in the real world.

The three main reasons are: it is too expensive to perform experiments on real systems, e.g. extensive drop tank separation analysis.it is too dangerous. The pilot can practice a dangerous maneuver before performing it in the plane, the system may not yet exist, i.e. the model will act as prototype that is evaluated and tested [13]. Other positive features that simulation provides are: variables not accessible in the real system can be observed in a simulation, it is easy to use and modify models and to change parameters and perform new simulations [14]. With system design optimization many variants can be evaluated, the time scale of the system may be extended or shortened. For example, a pressure peak can be observed in detail or a flight of several hours can be simulated in minutes. For aircraft it is common to use the three coordinate systems, body axes, stability axes and wind axes[2].

II. MATERIAL AND METHOD:

In this research paper aircraft is modeled for obtaining state space by taking three variables into consideration as illustrated in figure 2.1.

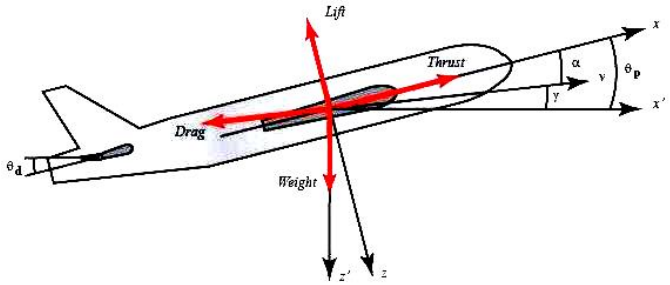


Fig 2. Aircraft Selected Variables [16]

The variables are as follows:

Table I: State Variables Selected

State Variable	Meaning
α	Angle of aircraft
q	Pitch rate
θ_p	Pitch angle

The input will be the elevator deflection and the output will be the pitch angle θ_p of the aircraft. Differential equation obtained for Boeing commercial aircraft [17] are given below:

$$\dot{\alpha} = 0.313\alpha - 56.7q + 0.232\theta_d \quad (2.1)$$

$$\dot{q} = 0.0139\alpha - 0.426q + 0.0203\theta_d \quad (2.2)$$

$$\dot{\theta}_p = 56.7q \quad (2.3)$$

The matrices obtained from the model are,

$$\begin{bmatrix} \dot{\alpha} \\ \dot{q} \\ \dot{\theta}_p \end{bmatrix} = \begin{bmatrix} -0.313 & 56.7 & 0 \\ -0.0139 & -0.426 & 0 \\ 0 & 56.7 & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ q \\ \theta_p \end{bmatrix} + \begin{bmatrix} 0.232 \\ 0.0203 \\ 0 \end{bmatrix} \theta_d \quad (2.4)$$

$$y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \alpha \\ q \\ \theta_p \end{bmatrix} \quad (2.5)$$

1) Importance of the State Space Model:

In Control Engineering, a state space representation is a mathematical model of a physical system as a set of input, output and state variables related by first order differential equations [5]. "State Space" refers to the space whose axes are the state variables. The state space analysis of control system is based on the modern theory which is applicable to all types of control systems including single input single output, multiple

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inputs and multiple output systems, linear and nonlinear systems, time varying and time invariant systems[8].

A set of state variables is the minimum set of variables, whose value at any instant, would enable the complete determination of the values of the state variables in the future. In fact, the state variables contain all the information about the past and the present, which is necessary for the prediction of their future values [7]. The algebraic equation expressing the output in terms of the state variables is called output equation. For multi-output systems, a separate output equation is needed to define each output. The state, input and output equations together provide a complete representation for the system described by the differential equation which is known as the state-space representation [12]. The state variables describe the present configuration of a system and can be used to determine the future response, given the excitation inputs and the equations describing the dynamics.

Most general continuous-time linear dynamical system has form,

$$\dot{x}(t)=A(t)x(t)+B(t)u(t) \quad (2.6)$$

$$y(t)=C(t)x(t)+D(t)u(t) \quad (2.7)$$

- $t \in \mathrm{R}$ denotes time
- $x(t) \in \mathrm{R}^{n}$ is the state vector
- $u(t) \in \mathrm{R}^{m}$ is the input or control variable
- $y(t) \in \mathrm{R}^{p}$ is the output
- $A(t) \in \mathrm{R}^{n\times n}$ is the dynamics matrix
- $B(t) \in \mathrm{R}^{n\times m}$ is the input matrix
- $C(t) \in \mathrm{R}^{p\times n}$ is the output or sensor matrix
- $D(t) \in \mathrm{R}^{p\times m}$ is the feed through matrix

x(t) is called the state of the system at time *(t)* because:

- Future output depends only on current state and future input
- Future output depends on past input only through the current state
- State summarizes effect of past inputs on future output - like the memory of the system

2) Simulink State Space Model:

The Simulink model developed for state space formulation is as follows. The function file for A matrix will be imported from Matlab.

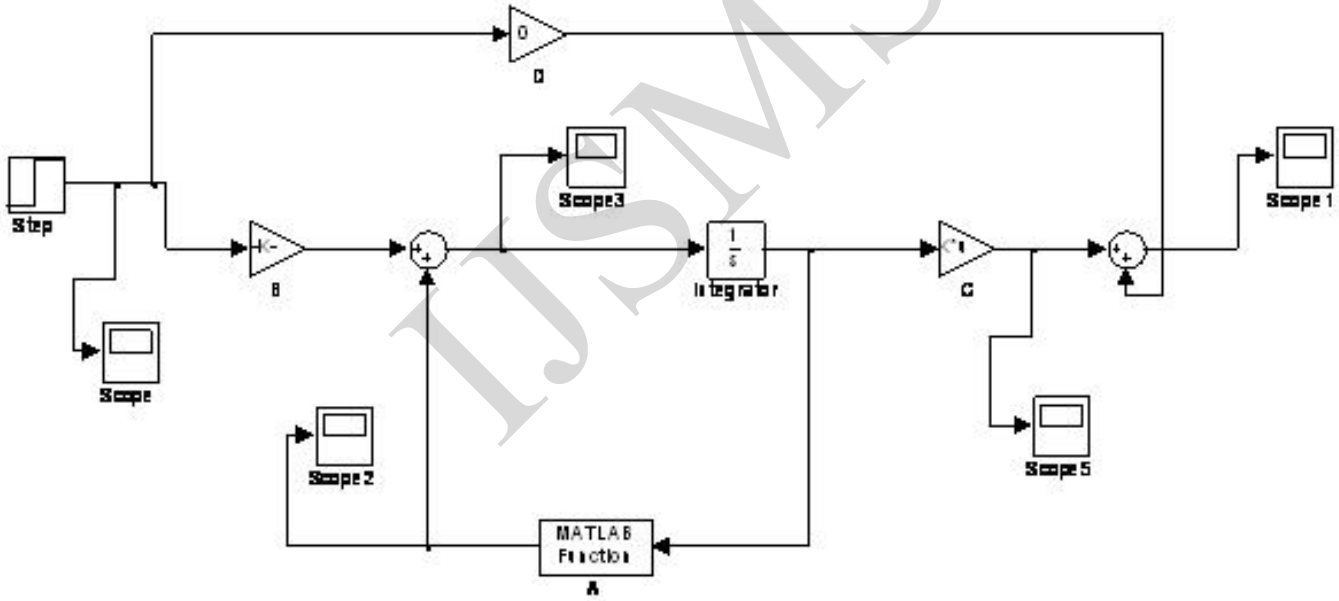


Fig 3. Simulink Model

The system is simulated and the following results are obtained.

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III. RESULTS AND DISCUSSION:

The system response is as given below:

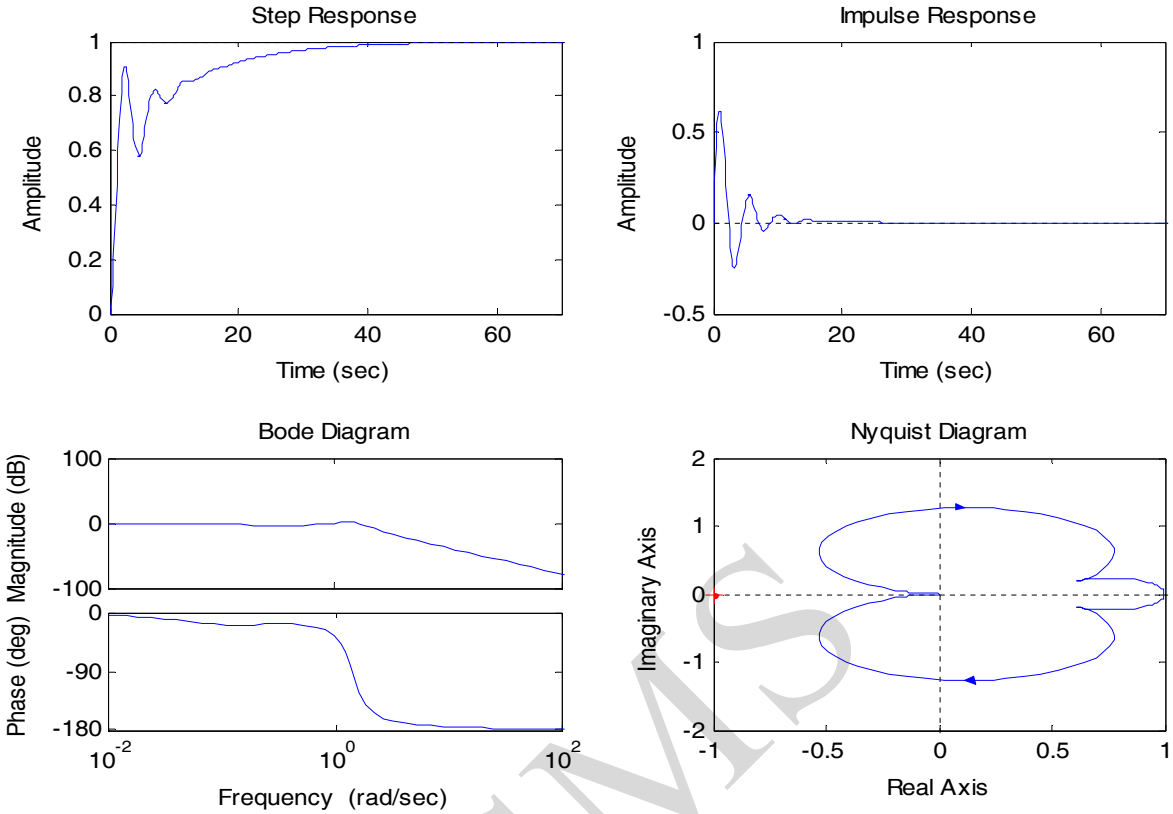


Fig 4. System Response

The system settles down at 31.6 sec as given below

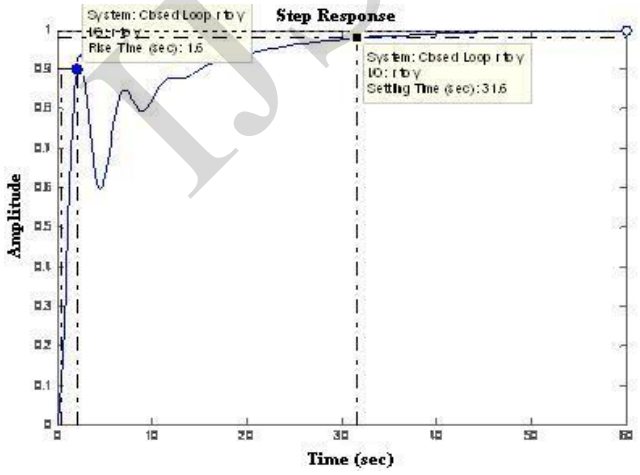


Fig 5. System Response to Command

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For improving the stability and decreasing the settling time to the nearly half value a PD controller is designed. The output after the addition of controller is as shown in figure 3.3.

In control theory, a controller is a device, historically using mechanical, hydraulic, pneumatic or electronic techniques often in combination, but more recently in the form of a microprocessor or computer, which monitors and physically alters the operating conditions of a given dynamical system [19]. Broadly speaking there are two types of controller; conventional and non conventional controllers. Conventional controllers can be designed for the system which can be represented by a mathematical model. The examples of conventional controllers are Proportional Derivative or PD controllers, Proportional Integral or PI Controllers or Proportional and Derivative controller PID controllers, The non-conventional controllers can be applied in which the mathematical model is not available for example fuzzy controllers[18].

In this paper a conventional controller is used. The PI controller is used to eliminate the steady state error from the system and also improves the response. A PI-Lead controller is a proportional gain in parallel with an integrator; both in series with a lead controller. The proportional gain provides fast error response[1]. The integrator drives the system to a 0 steady-state error. It has a pole at origin and a zero at a certain defined location. In contrast a PD controller is used to change the transient characteristics of the system. The aim of using P-D controller is to increase the stability of the system by improving control since it has an ability to predict the future error of the system response. In order to avoid effects of the sudden change in the value of the error signal, the derivative is taken from the output response of the system variable instead of the error signal. Therefore, D mode is designed to be proportional to the change of the output variable to prevent the sudden changes occurring in the control output resulting from sudden changes in the error signal. In addition D directly amplifies process noise therefore D-only control is not used. Derivative action is usually used to improve transient response of the closed loop system.

Only D control is not used because it amplifies high frequency noise which is never desired. Derivative action decreases rise time and oscillations. However, it does not have any effect on steady state performance of the closed loop. The discussion above indicates that with P-D control, steady state error is still non-zero. Derivative control is usually used to decrease oscillations in closed loop system outputs. The third one is the PID controller having all proportional derivative and integral actions. In this paper we targeted to reduce the settling time to half without producing any significant overshoot. For this purpose a PD is inserted along with the system to achieve the required response.

$$Z_c = -1.96 \quad (3.1)$$

(Controlled Zero added)

$$K_p = 3.52 \quad (3.2)$$

(Proportional Gain)

$$P_D = -1.96 \quad (3.3)$$

(Derivative Gain)

$$K_D\left(S + \frac{K_p}{K_D}\right) \quad (3.4)$$

$$\frac{K_p}{K_D} = 1.96 \quad (3.5)$$

$$\Rightarrow \frac{3.52}{K_D} = 1.96 \quad (3.6)$$

$$K_D = \frac{3.52}{1.96} \quad (3.7)$$

$$K_D = 1.7959 \quad (3.8)$$

The P_D attached to A matrix is,

$$1.7959(S + 1.96) \quad (3.9)$$

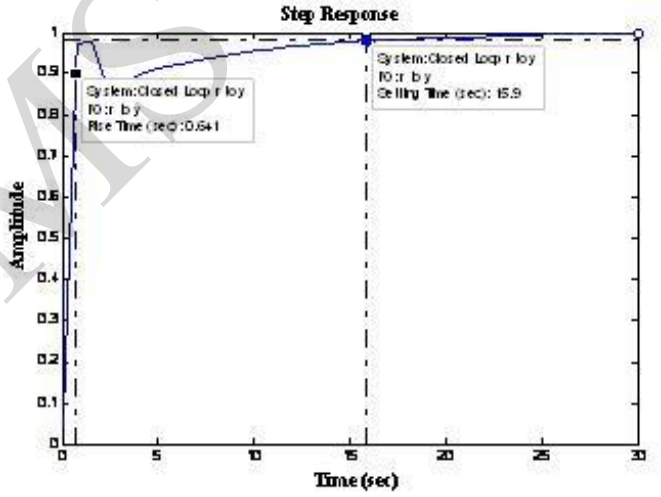


Fig 6. System Response with added Controller

IV. CONCLUSION:

Today's aircraft designs rely heavily on automatic control system to monitor and control many of aircraft's subsystem. The development of automatic control system has played an important role in the growth of civil and military aviation. For this situation an autopilot is designed that controls the pitch of aircraft that can be used by the flight crew to lessen their workload during cruising and help them land their aircraft during adverse weather condition in the real situation.

Aircraft development is crucial to understand and evaluate behavior, performance, safety and other aspects of the systems before and after they are physically available for testing. Simulation models are used to gain knowledge in order to make

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decisions at all development stages. In this paper the model that is used has some assumptions such as the aircraft is taken in steady cruise at constant altitude and velocity, balancing the thrust, drag, weight and lift forces in X and Y direction, secondly change in pitch angle will not change the speed of the aircraft under any circumstances and lastly he input will be the elevation differential θ_d and the output will be the pitch angle θ_p of the aircraft. For aircraft it is common to use the three coordinate systems, body axes, stability axes and wind axes. We have worked only on stability axis and emphasized on the fast settling of the system without significant overshoots. The state space model using Simulink is successfully formulated and simulated with and without controller. The angle of aircraft, pitch rate and pitch angle are selected as state variables.

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