Group Replacement Model Using Hidden Markov Model

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***Abstract*- Group replacement model is applicable to the items that fail completely on usage and the result is group replacement age for the entire group of items in the system irrespective of whether they are functioning or not. Generally Industrial items /equipment get worn with time and usage and it functions with decreasing efficiency. The increasing repair and maintenance cost demands the replacement of items. In the current model an intermediate state called repairable breakdown state in between functioning and irreparable breakdown state is considered and the Hidden Markov Model (HMM) is made use of in generating the probabilities of items falling in different states. Replacement decision is made considering money value. The optimal replacement age is the time bucket 't' at which the average cost of the individual replacement, repair and the cost of the group of items is minimum.**

***Index Terms* - GroupReplacement, HMM**

I. INTRODUCTION

The replacement problems are concern with the situation that arise when the efficiency of item decreases, failure or breakdown occurs. The decrease of efficiency or breakdown may be either gradual or sudden. The situation which demands the replacement of items are:

1. The old item has become inefficient or require expensive maintenance
2. The old item has failed due to accident or other wise and does not work at all, or the old item is expected to fail shortly
3. A better design of equipment has been developed or due to obsolescence

Group replacement is concerned with those items that either work or fail completely. It often happens that a system contains a large number of identical low cost items that are increasingly liable to failure with age. In such cases, there is a set-up cost for replacement that is independent of the number replaced and it may be advantageous to replace all items at fixed intervals. Such a policy is called *group replacement* and is particularly attractive when the value of any individual item is so small that the cost of keeping records of individual ages cannot be justified.

II. HIDDEN MARKOV MODEL (HMM)

Hidden Markov Model is a doubly stochastic process with an underlying stochastic process that is not observable (Hidden in nature), but can only be observed through another set of stochastic process that produce the sequence of observed symbols. HMM are an extension of Markov chains. A Markov chain is a random process of discrete valued variable involving a number of states linked by a number of possible transitions.

Nomenclature of HMM

Various parameters that constitute the structure of HMM for the length of observation sequence 'T' are

1) N: Finite number of states that are not visible (hidden) and are represented as

$S = \{S_1, S_2, S_3, \ldots S_N\}$

2) M: Number of distinct observations symbols(states) per hidden state. Individual symbols are represented by $V = \{V_1, V_2, V_3, \ldots V_M\}$ once the each hidden transition is made, an observation(visible) output state is generated according to a probability distribution which depends on the parent state.

3) A: Hidden state transition probability matrix

$$A = \{a_{ij}\} = \begin{Bmatrix} a_{11} & a_{12} & - & - & a_{1N} \\ a_{21} & a_{22} & - & - & a_{2N} \\ - & - & - & - & - \\ - & - & - & - & - \\ a_{N1} & a_{N2} & - & - & a_{NN} \end{Bmatrix}$$

$$a_{ij} = P[q_t = S_j \mid q_{t-1} = S_i] \qquad 1 \le i, j \le N$$

Where q_t denotes the actual hidden states at time 't' ,a_{ij} is the probability of moving from state S_i to S_j at time 't' and $\sum a_{ij} = 1$ for all 'i'

4) B: Observation symbol(visible states) probability distribution or output emission probability

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$B = \{ b_{jk} \} = P[o_t = V_m \mid q_t = S_j]$
$1 \leqslant k \leqslant M,\ 1 \leqslant j \leqslant N$

where o_t is the observation at time 't' and 'j' is state. After each transition is made a symbol will come out based on the output probability distribution which depends on the present state. $\sum b_{jk} = 1$ for all 'j'

$$B = \{ b_{jk} \} = \begin{Bmatrix} b_{11} & b_{12} & - & - & b_{1M} \\ b_{21} & b_{22} & - & - & b_{2M} \\ - & - & - & - & - \\ - & - & - & - & - \\ b_{N1} & b_{N2} & - & - & b_{NM} \end{Bmatrix}$$

5) π: Initial state distribution

$\pi = [\pi i]$ in which $\pi j = P(qi = Si),\ 1 \leq i \leq N$

πj is the initial probability matrix which stores the probability of the system. Starting at state 'i' in an observation. It is the probability of being in state 'i' at t = 1. The complete representation of HMM is made as $\lambda = (A, B, \pi)$

III. DEVELOPMENT OF MODEL

Concept of Hidden Markov Model is adopted for group replacement policy. The assumptions involved in development of model are:

- In group replacement decisions, in addition to the functioning state and failure state, an intermediate state called repairable breakdown state is introduced.
- This model can be applied to a system which consists of N items in three different states and to find out the group replacement period. Ex: For maintenance of cutting tools in manufacturing industry
- Repair cost or rectification cost of any item is constant.
- The item has three hidden states and three observable states.
- The failure of items is assumed to follow Weibull distribution

Notations

N_1 = Total number of items.
C_1 = Individual replacement cost per item.
C_2 = Group replacement cost per item.
C_3 = Rectification cost of the item.
r_n = Nominal interest rate assumed to constant during the life span of the asset,
ν = Present worth factor = $1 / (1 + r_n)$
X_0^I = Proportion of items in functional state.
X_0^{II} = Proportion of items in repairable breakdown state.
X_0^{III} = Proportion of items in irreparable breakdown state.
X_i^I = Probability of items in functional state in i^{th}period.
X_i^{II} = Probability of items in repairable breakdown state in i^{th}period.
X_i^{III} = Probability of items in irreparable breakdown state in i^{th} period.
a_{jk} = Probability of items transitioned from j^{th} state to k^{th} state
A(t)= Average cost per period in t^{th} group replacement policy.
A = Transition probability matrix

Transition probability matrix or the generator of Hidden Markov Process can be represented as

$$\mathbf{A} = \begin{array}{c} \\ I \\ II \\ III \end{array} \begin{array}{c} \begin{array}{ccc} I & II & III \end{array} \\ \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \end{array}$$

I = Functional State
II = Repairable breakdown state
III = Irreparable breakdown state

The considered observable states are Good, Moderate, Worst

Probability of items in different states can be computed as = (probability of items in different states in initial period) * A,

where n =1,2,3…

$\pi_{i+1} = \pi_i * A \qquad i = 0,1,2 \ldots n-1$

Where $\pi_i = [\, X_i^I X_i^{II} X_i^{III} \,]$

$[\, X_1^I\ X_1^{II}\ X_1^{III} \,] = [\, X_0^I\ X_0^{II}\ X_0^{III} \,]\ A$, Here n = 1
...
$[\, X_i^I X_i^{II} X_i^{III} \,] = [\, X_0^I\ X_0^{II}\ X_0^{III} \,]\ A$, Here n = i

Using TPMs, probabilities of items falling in different states i.e. $X_i^I, X_i^{II}, X_i^{III}, X_i^{IV}$ in i^{th} period are to be calculated. By using MATLAB Software probabilities of items falling in different states for future periods are calculated.

Number of individual replacement

1^{st}period, $f_1 = N_1\ X_1^{III}$
2^{nd}period , $f_2 = N_1\ X_2^{III} + f_1\ X_1^{III}$
3^{rd}period , $f_3 = N_1\ X_3^{III} + f_1\ X_2^{III} + f_2\ X_1^{III}$
4^{th}period , $f_4 = N_1\ X_4^{III} + f_1\ X_3^{III} + f_2\ X_2^{III} + f_3\ X_1^{III}$

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Number of repairable breakdowns

1^{st}period, $g_1 = N\ X_1^{II}$
2^{nd}period , $g_2 = N\ X_2^{II} + g_1\ X_1^{II}$
3^{rd}period , $g_3 = N\ X_3^{II} + g_1\ X_2^{II} + g_2\ X_1^{II}$
4^{th}period , $g_4 = N\ X_4^{II} + g_1\ X_3^{II} + g_2\ X_2^{II} + g_3\ X_1^{II}$
… … … … … …

Total cost up to 'n' periods

$F(t) = NC_2 + C_1 [f_1 + f_2 \nu + f_3 \nu^2 + \ldots\ldots f_n \nu^{n-1}] + C_3 [g_1 + g_2 \nu + g_3 \nu^2 + \ldots\ldots g_n \nu^{n-1}]$

Average cost per period

$A(t) = F(t) / (\Sigma \nu^{n-1})$

Maintenance costs in $(n+1)^{th}$ period $= R(n+1)$
$= C_1 [f_1 + f_2 \nu + \ldots f_n \nu^{n-1} + f_{n+1} \nu^n] + C_3 [g_1 + g_2 \nu + \ldots g_n \nu^{n-1} + g_{n+1} \nu^n]$

$R(t+1) > A(t)$ is equivalent to $A(t+1) > A(t)$

Sum of individual replacements and repair costs in (t+1) period should be greater than the average cost in 't' period, to group replace in t^{th} period.

IV. CASE STUDY

This model can be applied for maintenance of cutting tools in a big manufacturing. The cutting tools have three hidden sates and three observable symbols. The initial states probabilities, Initial state transition matrix and emission matrix are given. The cutting tool will have

C_1 = Rs.6000; C_2 = Rs. 4000; C_3 = Rs.1500; N_1 = 600; r_n = 20%

The generator for Hidden Markov Process can be given as

$$[\ X_0^I\ \ X_0^{II}\ \ X_0^{III}\ ] \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$= \begin{bmatrix} 0.667 & 0.167 & 0.167 \end{bmatrix} \begin{bmatrix} 0.75 & 0.125 & 0.125 \\ 0.6 & 0.1 & 0.3 \\ 0 & 0 & 1 \end{bmatrix}$$

Similarly $[X_i^I X_i^{II} X_i^{III}\ ] = [\ X_0^I\ X_0^{II}\ X_0^{III}\ ]\ A$, Here n = i

MATLAB Program

```
aij = [0.75 0.125 0.125; 0.6 0.1 0.3; 0 0 1] ;
(Initial Transition probability matrix of states)
bjk = [ 0.8 0.15 0.05; 0.5 0.3 0.2; 0.1 0.3 0.6];
(Initial Emission matrix)
x1 = floor(random('Weibull',2.8,20,1,10)) ;
x2 = [x1,x1,x1,x1,x1,x1];
[uaij,ubjk]= hmmtrain(x2,aij,bjk);
a=[0.667 0.166 0.167];
(Initial probabilities of states)
Uaij
b=a;
for i=1:10
i
 b=b*uaij
i=i+1;
end
```

OUTPUT

Modified Transition probability matrix of states

```
uaij =   0.3750   0.1875   0.4375
         0.2000   0.1000   0.7000
         0        0        1.0000
```

Probabilities of states for 10 periods

```
For i = 1,   b = 0.2833   0.1417   0.5750
Fori = 2,    b = 0.1346   0.0673   0.7981
Fori = 3,    b = 0.0639   0.0320   0.9041
Fori = 4,    b = 0.0304   0.0152   0.9545
Fori = 5,    b = 0.0144   0.0072   0.9784
For i = 6,   b = 0.0069   0.0034   0.9897
Fori = 7,    b = 0.0033   0.0016   0.9951
For i = 8,   b = 0.0015   0.0008   0.9977
For i = 9,   b = 0.0007   0.0004   0.9989
For i =10,   b = 0.0003   0.0002   0.9995
```

Table I: Replacement Decision

1	2	3	4	5
Period **t**	Present worth factor **$\nu = 1/(1 + r_t)$**	Discount factor **ν^{n-1}**	Dividing discount factor **$\Sigma\nu^{n-1}$**	Individual Replacement **$f_i \times C_1 \times \nu^{n-1}$** (Rs)
1	0.833	1	1	1019880.0000
2	0.833	0.833	1.833	644318.8393
3	0.833	0.694	2.527	406993.6161
4	0.833	0.578	3.105	257130.5511
5	0.833	0.482	3.587	162547.6072
6	0.833	0.4010	3.988	102628.6153
7	0.833	0.3340	4.322	64865.3098
8	0.833	0.2790	4.601	80345.4234
9	**0.833**	**0.2320**	**4.833**	**122577.7072**
10	0.833	0.1930	5.026	409882.2879

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6	7 = (5 + 6)	8	9
Minor Repairable Replacement $g_i \times C_3 \times v^{n-1}$ (Rs)	Maintenance Cost R_i (Rs)	$\sum R_i$ (Rs)	Group Replacement $N \times C_2 \times v^{n-1}$ (Rs)
127530.0000	1147410.0000	1147410.0000	2400000.0000
65507.9538	709826.7932	1857236.7932	1999200.0000
33677.1820	440670.7981	2297907.5913	1665600.0000
17299.3445	274429.8956	2572337.4869	1387200.0000
8888.9545	171436.5617	2743774.0485	1156800.0000
4552.9014	107181.5168	2850955.5653	962400.0000
2333.0374	67198.3472	2918153.9125	801600.0000
2942.8061	83288.2296	3001442.1420	669600.0000
4514.3269	**127092.0342**	**3128534.1762**	**556800.0000**
7359.3462	417241.6341	3545775.8103	463200.0000

10 = (8 + 9)	11 = (10 / 4)
Total cost **TC(t)** (Rs)	Average Cost **A(t)** (Rs)
3547410.0000	3547410.0000
3856436.7932	2103893.5042
3963507.5913	1568463.6293
3959537.4869	1275213.3613
3900574.0485	1087419.5842
3813355.5653	956207.5139
3719753.9125	860655.6947
3671042.1420	797879.1876
3685334.1762	**762535.5217**
4008975.8103	797647.3956

V. CONCLUSIONS

By observing the average costs for various periods, it is found that the average cost is gradually decreasing up to ninth period and for the next period it is increasing suddenly.

Economics can be achieved if the group replacement is done at the end of ninth period. Since the model is developed using Hidden Markov Model better probabilities of various states (Functional, Repairable, and Breakdown) are obtained. Unlike conventional group replacement models this model permits an intermediate state like repairable breakdown stage of item.

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