Measuring the technical efficiency of local banks in UAE using rough bi-level linear programming technique

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***Abstract*–The aim of this paper is to use a bi-level linear programming technique with rough parameters in the constraints, for measuring the technical efficiency of local banks in UAE and Egypt, while the proposed linear objective functions will be maximized for different goals. Based on Dauer's & Krueger's goal programming method, the described approach was developed to deal with the bi-level decision-making problem. The concept of tolerance membership function together was used to generate the optimal solution for the problem under investigation. Also an auxiliary problem is discussed to illustrate the functionality of the proposed approach.**

***Index Terms*– Bi –level programming, rough programming, goal programming, banking efficiency, auxiliary problem, tolerance membership, branch and bound**

I. INTRODUCTION

A ratio used to calculate abank's efficiency. Not al l banks calculate the efficiency ratio the same way.

The ratio can be calculated one of four ways:

1. noninterest expense divided by total revenue less interest expense,
2. noninterest expense divided by net interest income before provision for loan los ses,
3. Noninterest expense divided by revenue, operating expenses divided by fee income plus tax equivalent net interest income.

In all four methods,an increase means the compan y is losing a larger percentage of its income to expenses.

If the efficiency ratio is getting lower, it is good for thebank and its shareholders. Also referred to as the overhead burden or overhead efficiency ratio.

Regardless of which ratio calculation method is used, the efficiency ratio's purpose is to evaluate the overhead structure of a financial institution. Banking is no different from any mature industry: Companies that survive keep their costs down.

The efficiency ratio measures how effectively a bank i s operating and how profitable it is.

Rough set theory [5] is an effective tool for data mining, and it has been the core problem in academic and application fields.

Many scholars gave many discussions under different background and also obtained many important research results [3].

Bi-level programming problem is formulated for a problem in which two decision makers make decisions successively [7].

The majority of research on bi-level decision making has centered on the linear BLP. A set of approaches and algorithms of linear bi-level programming, such as the well-known Kuhn–Tucker approach, Kth-best approach, Branch-and-bound algorithm and genetic algorithm have been widely used [9].

Diverse forms of rough sets have been proposed and studied with different binary relations. Moreover, some of them are investigated with addition topological or algebraic structures [4,8].

The goal programming (GP) model is useful for decision makers to consider several objectives in finding a set of acceptable solutions simultaneously. Since only partial information can possibly be obtained, precisely determining the goal value of each objective might be difficult for decision makers; the main studies that incorporate uncertainty and imprecision into the GP are [6].

In [1] Emam proposed a bi-level integer non-linear programming problem with linear or non-linear constraints, and in which the non-linear objective function at each level are to maximized. It proposed a two planner integer model and a solution method for solving this problem. Emam proposed an interactive approach for solving bi-level integer multi-objective fractional programming problem [2].

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In [10] Xu and Yao discussed a class of linear multi-objective programming problems with random rough coefficients and gave a crisp equivalent model .

In[7] Saraj and Sadeghi presented a fuzzy goal programming (FGP) technique for solving Quadratic Bi-Level Fractional Multi-Objective Programming (QBL-FMOP) Problem was developed.

There is a bank has two branches, one of them (the main branch) in AUE (Leader) and the other in Egypt (Follower) , these two branches have objectives . Let N1 Increase the amount of deposits

N2 Increase the amount of loans. And they Invest in financial bonds X1 , X2 .It's easy to know that the main branch who put the strategy and goals of the follower branch.

II. Problem Formulation and Solution Concept

Let $x_i \in R, (i = 1,2)$ be a vector variables indicating the first decision level's choice and the second decision level's choice,$(i = 1,2)$.

Let $H_i: R^2 \to R^{N_i}, (i = 1,2)$ be the first level objective functions, and the second level objective functions, respectively. Let the (FLDM) the first level decision maker or the AUE branch and (SLDM) the second level decision maker or the Egypt branch have N_1 and N_2 objective function, respectively. AUE branch sets his goals and the Egypt branch first.

Therefore, the bi- level multi-objective linear programming problem with definite goals contains rough parameters in constraints may be stated as follows:

[FLDM]

$$\max_{x_1} H_1(x_1, x_2) = \max_{x_1}(h_{11}, \dots, h_{1N_1}), \quad (1)$$

Where x_2 solves

[SLDM]

$$\max_{x_2} H_2(x_1, x_2) = \max_{x_2}(h_{21}, \dots, h_{2N_2}), \quad (2)$$

Subject to

$$G = \{(\xi x_1, \xi x_2) | g_i(\xi x_1, \xi x_2) \le y, i = 1,2,\dots m, (x1, x2) \ge 0. \quad (3)$$

Where

$$f_{kj} = \sum_{j=1}^{n} \xi_{kj} x_j \ , i = 1,2, \dots, n_k.$$

Gis the bi-level linear constraint set contains rough parameters.$H_1$and$H_2$are linear functions with definite goals.

Now, going back to the bi- level multi-objective linear programming problem contains rough parameters in constraints, We can write an associated goal programming for this problem with$(N_1 + N_2)$ goals as follows:-

[FLDM]

Achieve $(h_{11}(x), \dots, h_{1N_1}(x)) = (k_{11}, \dots k_{1N_1})$, (4)

Where x_2 solves

[SLDM]

Achieve$(h_{21}(x), \dots, h_{2N_2}(x)) = (k_{21}, \dots k_{2N_2})$,(5)

Subject to

$$x \in G.$$

Where k_{1N_1}, k_{2N_2} are scalars and represent the aspiration levels associated with the objectives of the First level decision maker and second level decision maker, respectively.

III. The Transformation of Random Rough Coefficient [10]

To convert the bi-level multi-objective linear programming problem with random rough coefficient in the constraints into the respective crisp equivalents for solving this trust probability constrains, this process is usually hard work for many cases but the transformation process is introduced in the following theorems.

Theorem 1

Suppose that random variables $\tilde{\tilde{e}}_{rj}(\lambda)$ and $\tilde{\tilde{b}}_r(\lambda)$ are characterized by $\tilde{\tilde{e}}_{rj}(\lambda) \sim \mathcal{N}(e_{rj}(\lambda), V_r^e), \tilde{\tilde{b}}_r(\lambda) \sim \mathcal{N}\left(b_r(\lambda), (\sigma_r^b)^2\right)$,wher e $e_{rj}(\lambda), b_r(\lambda)$ are rough variables, and $V_r^e, (\sigma_r^b)^2$are positive definite covariance's. By Theorem 1, we have that $e_r(\lambda)^T x, b_r(\lambda)$are rough variables; then $e_r(\lambda)^T x - b_r(\lambda) = [(a,b),(c,d)](c \le a \le b \le d)$ is also a rough variable. We assume that it is characterized by the following trust measure function:

$$\mathrm{Tr}\{e_r(\lambda)^T x - b_r(\lambda) \le t\} = \begin{cases} 0 & \text{if } t \le c, \\ \dfrac{t-c}{2(d-c)} & \text{if } c \le t \le a, \\ \dfrac{1}{2}\left(\dfrac{t-c}{d-c} + \dfrac{t-a}{b-a}\right) & \text{if } a \le t \le b, \\ \dfrac{1}{2}\left(\dfrac{t-c}{d-c} + 1\right) & \text{if } b \le t \le d, \\ 1 & \text{if } d \le t. \end{cases}$$

Where$\left(e_{rj}(\lambda)\right)_{nx1} = \left(e_{r1}(\lambda), e_{r2}(\lambda), \dots, e_{rn}(\lambda)\right)^T$.

Then, we have that $\mathrm{Tr}\left\{\lambda \middle| \Pr\left\{\tilde{\tilde{e}}_r(\lambda)^T x \le \tilde{\tilde{b}}_r(\lambda)\right\} \ge \theta_r\right\} \ge \eta_r$ if and only if

$$\begin{cases} a \ge M \ge c + 2(d-c)\eta_r \\ b \ge M \ge \dfrac{2\eta_r(d-c)(b-a) + C(b-a) + a(d-c)}{d-c+b-a} \\ d \ge M \ge (2\eta_r - 1)(d-c) + c \\ M \ge d \end{cases}$$

To proof theorems1 above, the reader is referred to [10].

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a. The equivalent crisp problem of bi-level rough linear problem

The equivalent bi-level multi-objective linear programming problem equivalent to the bi- level multi-objective linear programming problem contains rough parameters in constraints with definite goals may be stated as follows:

[FLDM]

$$\max_{x_1} H_1(x_1, x_2) = \max_{x_1}(h_{11}, \dots, h_{1N_1}), \quad (6)$$

Where x_2 solves

[SLDM]

$$\max_{x_2} H_2(x_1, x_2) = \max_{x_2}(h_{21}, \dots, h_{2N_2}), \quad (7)$$

Subject to

$$G = \{(x_1, x_2) | g_i(x_1, x_2) \leq 0, i = 1,2, \dots m, (x_1, x_2) \geq 0. \quad (8)$$

Where h_1, h_2 are the objective functions of the first level decision maker (FLDM), and second level decision maker (SLDM).

Definition 1:

For any $x_1 (x_1 \in G_1 = \{x_1 | (x_1, x_2) \in G\})$ achieves the first level decision maker goals with under-attainment or over-attainment , if the decision-making variable $x_2 (x_2 \in G_2 = \{x_2 | (x_1, x_2) \in G_1\})$ achieves the second level decision maker goals with under-attainment or over-attainment, then (x_1, x_2) is a feasible solution of the rough goal bi-Level multi-objective linear programming problem.

Definition 2:

If (x_1^*, x_2^*) is a feasible solution of the rough goal bi-Level multi-objective linear programming problem, such that the first level decision maker achieves all goals; so (x_1^*, x_2^*) is the Pareto optimal solution of the rough goal bi-level multi-objective linear programming problem.

IV. A GOAL APPROACH FOR THE BI- LEVEL MULTI-OBJECTIVE LINEAR PROGRAMMING PROBLEM

To solve the bi- level multi-objective linear programming problem with definite goals, one first get the optimal solution of the FLDM with definite goals, and the SLDM should get his optimal solution with definite goals, as follows:-

a. The first level decision maker

First, the FLDM solves the following problem:

$$\text{Achieve } (h_{11}(x), \dots, h_{1N_1}(x)) = (k_{11}, \dots k_{1N_1}), \quad (9)$$

Subject to

$$x \in G.$$

Where $k_{11}, \dots, k_{1N_1}$ are scalars, and represent the aspiration levels associated with the objectives, $h_{11}, \dots, h_{1N_1}$, respectively.

We consider the following bi-level multi-objective linear programming problem associated to the first goal as:

$$P_{11}: \quad \text{Minimize } D_{11} = d_{11}^- + d_{11}^+, \quad (10)$$

Subject to

$$h_{11}(x) + d_{11}^- - d_{11}^+ = k_{11},$$
$$x \in G,$$
$$d_{11}^-, d_{11}^+ \geq 0.$$

Where d_{11}^- and d_{11}^+ are the under-attainment and over-attainment, respectively, of the first goal and $d_{11}^- X\, d_{11}^+ = 0$.

Then the attainment problem associated with the second goal is equivalent to the optimization problem P_{12} , where:

$$P_{12}: \quad \text{Minimize } D_{12} = d_{12}^- + d_{12}^+, \quad (11)$$

Subject to

$$h_{12}(x) + d_{12}^- - d_{12}^+ = k_{12},$$
$$h_{11}(x) + d_{11}^- - d_{11}^+ = k_{11},$$
$$d_{11}^- + d_{11}^+ = D_{11}^*,$$
$$x \in G,$$
$$d_{1t}^- + d_{1t}^+ \geq 0.$$

The optimal solution of the linear goal programming model is given by $x^* = (x_1^F, x_1^F)$.

b. The second level decision maker

Second, in the same way, the SLDM independently solves:

$$\text{Achieve}(h_{21}(x), \dots, h_{2N_2}(x)) = (k_{21}, \dots k_{2N_2}), \quad (12)$$

Subject to

$$x \in G.$$

Where $k_{21}, \dots, k_{2N_2}$ are scalars, and represent the aspiration levels associated with the objectives, $h_{21}, \dots, h_{2N_2}$, respectively.

The SLDM will do the same action as the FLDM till he obtain his optimal solution $x^* = (x_1^S, x_1^S)$.

V. FUZZY APPROACH OF BI-LEVEL LINEAR PROGRAMMING WITH ROUGH PARAMETERS PROBLEM

Now the solution of the AUE branch and Egypt branch are disclosed. But, two solutions are usually different because of nature between two levels goals. The AUE branch knows that using the optimal decisions x_1^F as a control factors for the EGYPT branch, is not practical. It is more reasonable to have some tolerance that gives the EGYPT branch an

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extent feasible region to search for his/her optimal solution, and reduce searching time or interactions.

In this way, the range of decision variable x_1 should be around x_1^F with maximum tolerance t_1 and the following membership function specify $x_1$as:

$$\mu(x_1) = \begin{cases} \frac{x_1-(x_1^F-t_1)}{t_1} x_1^F - t_1 \le x_1 \le x_1^F, \\ \frac{-x_1+(x_1^F+t_1)}{t_1} x_1^F \le x_1 \le x_1^F - t_1, \end{cases} \quad (13)$$

Where X_1^F is the most preferred solution; the $(X_1^F - t_1)$and$(X_1^F + t_1)$ are the worst acceptable decision; and that satisfaction is linearly increasing with the interval of $\lfloor X_1^F - t_1, X_1 \rfloor$ and linearly decreasing with$\lfloor X_1, X_1^F - t_1 \rfloor$, and other decision are not acceptable.

First, the FLDM goals may reasonably consider $h_1 \ge h_1^F$ is absolutely acceptable and $h_1 < \grave{h_1}$ is absolutely unacceptable, and that the preference with $[\grave{h_1}, h_1^F]$ is linearly increasing. This due to the fact that the SLDM obtained the optimum at(x_1^S, x_2^S), which in turn provides the FLDM the objective function values$\grave{h_1}$, make any $h_1 < \grave{h_1} = h_1(x_1^S, x_2^S)$ unattractive in practice.

The following membership functions of the FLDM can be stated as:

$$\grave{\mu}[h_1(x)] = \begin{cases} 1 & \text{if } h_1(x) > h_1^F, \\ \frac{h_1(x)-\grave{h_1}}{h_1^F-\grave{h_1}} & \text{if } \grave{h_1} \le h_1(x) \le h_1^F, \\ 0 & \text{if } \grave{h_1} \ge h_1(x). \end{cases} \quad (14)$$

Second, the SLDM goals may reasonably consider the $h_2 \ge h_2^S$ is absolutely acceptable and $h_2 < \grave{h_2}$ is absolutely unacceptable, and that the preference with $[\grave{h_2}, h_2^S]$ is linearly increasing. In this way, the SLDM has the following membership functions for his/her goal:

$$\grave{\mu}[h_2(x)] = \begin{cases} 1 & \text{if } h_2(x) > h_2^S, \\ \frac{h_2(x)-\grave{h_2}}{h_2^S-\grave{h_2}} & \text{if } \grave{h_2} \le h_2(x) \le h_2^S, \\ 0 & \text{if } \grave{h_2} \ge h_2(x). \end{cases} \quad (15)$$

Finally, in order to generate the satisfactory solution, which is also a Pareto optimal solution with overall satisfaction for all decision-makers, we can solve the following Tchebycheff problem.

$$\max \delta, \quad (16)$$

Subject to

$$\frac{x_1 - (x_1^F - t_1)}{t_1} \ge \delta,$$

$$\frac{-x_1 + (x_1^F + t_1)}{t_1} \ge \delta,$$

$$\grave{\mu}[h_1(x)] \ge \delta,$$

$$\grave{\mu}[h_2(x)] \ge \delta,$$

$$x \in G,$$

$$t_i > 0\,, \delta \in [0,1].$$

Where δ is the over all satisfaction.

If the FLDM is satisfied with solution then satisfactory solution is reached. Otherwise, he/she should provide new membership function for the control variable and objectives to the SLDM, until a satisfactory solution is reached.

VI. Numerical Example

This example aims at determining the extent to which local banks held banking efficiency. The banking sector is considered sensitive and important in any society, as it is the basement and the pillar of success or failure of any economy. Hence, measuring efficiency of resources exploitation of this sector is considered necessary, to rationalize and direct the process of the banking industry for every country.

The standard analytical method has been used in this study by using the solution method for bi-level

Multi-objective linear programming problem under random rough coefficient in constraints can be written as:

The formula varies, but the most common one is:

Bank Efficiency Ratio = Expenses* / Revenue.

*not including interest expense.

As we said earlier, the formulas vary but the idea is to look at costs as a percentage of revenue. Costs include salaries, rent and administrative expenses. Interest expenses are usually excluded because they are investing decisions, not operational decisions

Revenue includes interest income and fee income, though some banks exclude their provision for loan losses from revenue or add their tax equivalent net interest income to revenue when calculating the efficiency ratio.

We have a Central main branch which is a first level decision maker for all branches named FLDM Located in Dubai, AUE And dependent branch in Cairo, Egypt named SLDM. The Main branch in UAE determines specific objectives which are increase properties Loans and industrial projects Loans.

The main branch target for it self 50 million $ for the first objective, and 20 millions $ for the second one. Then target for Egypt branch 10 millions $ for the first objectives and 18 millions $ for the second objectives. All these objectives determined in a rough environment. In this environment it is difficult to determine the market fluctuation. Which make the solution of this problem impossible with our traditional ways . So bi-level multi-objective linear programming problem under random rough coefficient in constraint is our choice to help us in solving this kind of problems.

Lets assume that the AUE branch has the following goals $h_1 = (50.9874, 20.9871)$, and the

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EGYPT branch has the following goals $h_2 = (11.9739,21.974)$ Overall satisfaction for both decisions makers.

[FLDM]

$\max_{x_1} h_{11}(x) = 5x_1 + x_2,$

$\max_{x_1} h_{12}(x) = 2x_1 + x_2,$

Where x_2 solves

[SLDM]

$\max_{x_2} h_{21}(x) = x_1 + 2x_2,$

$\max_{x_2} h_{22}(x) = 2x_1 + 2x_2,$

Subject to

$\xi_1 x_1 + \xi_2 x_2 <= 45,$

$\xi_3 x_1 + \xi_4 x_2 <= 30,$

$x_1, x_2 \geq 0.$

Assume that the rough parameters are defines as:

$\xi_1 \sim \mathcal{N}(\rho_1, 1)$, with $\rho_1 = ([2,3],[1,4])$,

$\xi_2 \sim \mathcal{N}(\rho_2, 4)$, with $\rho_2 = ([1,2],[1,3])$,

$\xi_3 \sim \mathcal{N}(\rho_3, 1)$, with $\rho_3 = ([1,2],[0,3])$,

$\xi_4 \sim \mathcal{N}(\rho_4, 2)$, with $\rho_4 = ([3,4],[2,5])$,

Let $\eta_r = 0.9$.

Now by using theorem 1, the equivalent crisp problem which equivalent to bi-Level multi-objective linear programming problem under rough parameters in constraints with definite goals, as follows:-[FLDM]

Achieve $h_{11} = (5x_1 + x_2) = k_{11}$,

Achieve $h_{12} = (2x_1 + x_2) = k_{12}$,

Where x_2 solves

[SLDM]

Achieve $h_{21} = (x_1 + 2x_2) = k_{21}$,

Achieve $h_{22} = (2x_1 + 2x_2) = k_{22}$.

Subject to

$x \in G = \{3.4x_1 + 2.6x_2 <= 45,$

$2.4x_1 + 4.4x_2 <= 30,$

$x_1 \geq 0, x_2 \geq 0 \geq 0\}.$

Then, calculating trust for every rough coefficients using trust measure function in theorem 1:

$\mathrm{Tr}\{\xi_1\} = 0.9, \mathrm{Tr}\{\xi_2\} = 0.9, \mathrm{Tr}\{\xi_3\} = 0.9, \mathrm{Tr}\{\xi_4\} = 0.9, \mathrm{Tr}\{\xi_5\} = 0.6, \mathrm{Tr}\{\xi_6\} = 0.9.$

So, with trust more than or equal $\propto$ is 0.6 the equivalent crisp problem which equivalent to bi-Level multi-objective linear programming problem under rough parameters in constraints.

Now, we can write an associated goal programming for this problem with ($N_1 + N_2$) goals as follows:-

[FLDM]

$\max_{x_1} h_1(x_1, x_2) = \max_{x_1} [5x_1 + x_2, 2x_1 + x_2],$

Where x_2 solves

[SLDM]

$\max_{x_2} h_2(x_1, x_2) = \max [x_1 + 2x_2, 2x_1 + 2x_2],$

Subject to

$x \in G = \{3.4x_1 + 2.6x_2 <= 45,$

$2.4x_1 + 4.4x_2 <= 30,$

$x_1 \geq 0, x_2 \geq 0 \geq 0\}.$

1- First, the FLDM solves his/her Problem as following:

Achieve $5x_1 + x_2 = k_{11}$,

Achieve $2x_1 + x_2 = k_{12}$,

Subject to

$x \in G.$

The aspiration levels of the goals are assumed to be $k_{11} = 50$, $k_{12} = 20$, respectively. Then, the optimization problem associated with the first goal is formulated as follows:

P_{11}: Minimize $D_{11} = d_{11}^- + d_{11}^+,$

Subject to

$5x_1 + x_2 + d_{11}^- - d_{11}^+ = k_{11},$

$x \in G,$

$d_{11}^-, d_{11}^+ \geq 0.$

The maximum degree of attainment of problem P_{11} is $D_{11}^* = 0.0002$ with the optimal solution $x_1 = (9.9998,0.0011)$ and $d_{11}^- = 0, d_{11}^+ = 0.0002$.

The attainment problem for goal 2 of the FLDM is equivalent to problem P_{12}, where:

P_{12}: Minimize $D_{12} = d_{12}^- + d_{12}^+$

Subject to

$2x_1 + x_2 + d_{12}^- - d_{12}^+ = k_{12},$

$5x_1 + x_2 + d_{11}^- - d_{11}^+ = 50,$

$d_{11}^- + d_{11}^+ = 0.0002,$

$x \in G,$

$d_{1t}^- + d_{1t}^+ \geq 0.$

Therefore, the optimal solution of the model P_{12} is $x_2 = (9.9999,0.0008), d_{11}^- = 0, d_{11}^+ = 0.0002, d_{12}^- = 0, d_{12}^+ = 0.0006$, so the optimal solution of the bi-level multi-objective linear goal programming model is given by x^* which will be the optimal solution of the FLDM $x^* = (x_1, x_2) = (9.9999,0.0008)$.

2- Second, the SLDM solves his/her Problem as following:

Achieve $x_1 + 2x_2 = k_{21}$,

Achieve $2x_1 + 2x_2 = k_{22}$,

Subject to

$x \in G.$

The aspiration levels of the goals are assumed to be $k_{21} = 10, k_{22} = 18$ respectively. Then, the optimization problem associated with the first goal is formulated as follows:

P_{21}: Minimize $D_{21} = d_{21}^- + d_{21}^+,$

Subject to

$x_1 + 2x_2 + d_{21}^- - d_{21}^+ = 10,$

$x \in G,$

$d_{21}^- + d_{21}^+ \geq 0.$

The maximum degree of attainment problem P_{21} is $D_{21}^* = 0$ with the optimal solution $x = (9.9987,0.0006)$ and $d_{21}^- = 0, d_{21}^+ = 0$.

The attainment problem for goal 2 of the SLDM is equivalent to problem P_{21}, where:

P_{22}: Minimize $D_{22} = d_{22}^- + d_{22}^+,$

Subject to

$2x_1 + 2x_2 + d_{22}^- - d_{22}^+ = 18,$

$x_1 + 2x_2 + d_{21}^- - d_{21}^+ = 10,$

$x \in G.$

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3- Finally, we assume the FLDM control decision $x_1^F = 9.9999$with the tolerance 1,the SLDM solves the following Tchebycheff problem as follows:

Whose, optimal solution is:$\left(d_{21}^F, d_{22}^F\right) = (10.0001,0.9869)$, $x_1^F = 0.9999$, $h_1 = (50.9874,20.9871)$,

And $h_2 = (11.9739,21.974)$Overall satisfaction for both decisions makers.

VII.CONCLUSION

This paper proposed a bi- level linear programming problem with rough parameters in constraints, the linear objective functions are to be maximized with different goals, the suggested approach in this paper was mainly based on the goal programming method of Dauer and Krueger to develop the optimal solution of the two- level decision- maker, then we used the concepts of tolerance membership function together with the branch and bound technique to generate the optimal solution for this problem.

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