A Mathematical Model for Weighted Multicriteria Assignment Problem: An Empirical Study

Burcu ADIGUZEL MERCANGOZ, PhD
Istanbul University Faculty of Transportation & Logistics
Istanbul, Turkey
burcua@istanbul.edu.tr

***Abstract*—Assignment is a key issue for the most planning process. It has always a big importance for the planning process. In most cases assignment is being done with more than one variable, taking in to consideration of both qualitative and quantitative variables. In this study a mathematical model for the weighted multicriteria assignment problem is developed and the results are displayed on a numerical example. The given numerical example is about to assign the faculty to the final exams of the courses. There are some constraints for the faculty according to their program. So they are given a matrix for the exams that they are available to be assigned. This study is unique that there has not been found a study with such this objective and these constraints. The problem examined is an integer programming with weighted objective to the best of my knowledge. A mathematical model is developed for the solution of this assignment problem. The effectiveness of the proposed modelis validated by its application to an empirical example dealing with weighted multiobjectiveassignment problem.**

Index: **Multicriteria Decision Analysis, Weighted Multicriteria Assignment Problem, Mathematical Model**

I. INTRODUCTİON

The assignment problem was first defined by G. Mongeas a transportation problem [1]. It is one of the first studied combinatorial optimization problems in the branch of optimization in mathematics. Dantzig showed that the assignment problem can be formulated as a linear programming problem at his study in 1951[12]. It has been a breakthrough in solving the assignment problem. Assignment problem may be used for assignment of workers to jobs, airplanes to the gates, teacher to class and so forth [3]. The main objective of assignment problem is to assign the right pair to the each task with the minimum total time to complete a set of tasks or the cost of the assignments or with the maximum skill ratings [10]. The standard assignment problem consists of assigning a number of tasks to an equal number of agents. Each agent is assigned to exactly one task and each task has exactly one agent assigned to perform it in such a way to minimize the overall cost of assigning agents to tasks [6]. However in some cases assignment problem can be with multi criteria. Multi Criteria Assignment problems usually do not have an optimal solution for all the criteria. That's the main difficulty of multicriteria assignment. Thus, some trade-off must be done among the different points of view to determine an acceptable solution [10]. The weighted assignment model represents problems with the following characteristics: A set of tasks must be divided among a set of agents, and only one agent must complete each task. Tasks may be completed at one of several predetermined levels. When an agent completes a task at some level, a resource possessed by the agent is consumed and a system disutility is incurred. The weighted assignment model may be used to determine the best assignment of tasks to agents and the task completion levels so as to minimize total system disutility while satisfying the agent resource constraints [8]. In this study it is aimed to solve a weighted multicriteria assignment problem by using integer programming. In the first part of the paper, relevant literature is searched and the methods used for the assignment problems are discussed. Subsequently, the improved model is given. Afterwards, the numerical example from the real world is solved with the improved model. Finally the computational results are given.

II. LITERATURE REVIEW

As the literature is searched, it can be seen that the assignment problem has been searched since the last four decades. Integer programing model with minimizing the objective function is widely used to solve the classical assignment problem [4;7]. However there is a limited range of literature on assigning with maximizing the workers' preferences. Most of the studies solve this problem with multicriteria assignment [9; 6] The Electre tri model is used to solve the multicriteria assignment problem [8], genetic algorithm (GA) is used to solve the multicriteriaassignment problem[2,5]. In this study the difference of the number of assignments to the faculties with the same title istried to be stabilized.

III. METHODOLOGY AND THE MODEL

Consider a set of faculties who are working in the faculty. In the exam week, there is a need of invigilators for performing the exams. Faculties are taking task as invigilators. The number of invigilator needed for each exam depends on the number of students having the exam so that all the exams need different number of faculty. The number of faculty needed for each exam is defined by "d_i". Faculties have four type of title as associate professor, assistant professor, PhD. research assistant and research assistant. According to their title, faculties' can be assigned to a different number of exams. Also the importance given to the preference of the faculties is differing according to their title. Weighting their preferences is used in the objective function. The entire faculties express their preference of the

IJARE

specific time for the exam on a sheet by giving coefficient to the desired time of having task. They gave `10` to express to willingness to be assigned to the exams. The value '-99' is used for not likely to be assigned to the exam and the value '-10' is used for the time that they rather not to have an exam.

The two-stage method is as follows:

1st Stage: Weighting the faculties' importance factor. The weight scores of each faculty member are determined by their title. Other weighting methodologies such as AHP can also be used however we used this for making the faculty as a part of this solution.

2ndStage: The objective function is determined as maximization. It is proposed to maximize the wish values of the faculties' number of assignment in the desired time for the faculties. Faculties with the same title are needed to have nearly same number of assignments, whether the faculty with the higher title has less constraints for the exam times, model assign more works to them, so that we solve this problem by giving limitations to their assignments.

The mathematical formulation of the weighted multicriteria assignment problem is as follows;

Model Notations

Sets

A = {1,..,H} set of faculties titled as an associate professor indexed by j

B = {H+1, …., S} set of faculties titled as an assistant professor indexed by j

C = {S+1,…,T} set of faculties titled as a phd research assistant indexed by j

D = {T+1,…,M} set of faculties titled as an research assistant indexed by j

N = {1,…, N} set of exams indexed by i

K = {1,……,K} set of weighting values of the faculty's preferences indexed by k

Model Parameters

i = the number of the exams i ={1,…..N}

j = the number of the faculty j ={1,…..M}

c_{ij} = The wish value of the faculty j on the time of the exam i. (According to their preferences `10` to express to willingness to be assigned to the exams. '-99' for not likely to be assigned to the exam and '-10' for the time that they rather not to have an exam.)

A_1 = minimum number of assignment for the faculties titled as an associate professor

A_2 = maximum number of assignment for the faculties titled as an associate professor

B_1 = minimum number of assignment for the faculties titled as an assistant professor

B_2 = maximum number of assignment for the faculties titled as an assistant professor

C_1 = minimum number of assignment for the faculties titled as a PhD research assistant

C_2 = maximum number of assignment for the faculties titled as a PhD research assistant

D_1 = minimum number of assignment for the faculties titled as a research assistant

D_2 = maximum number of assignment for the faculties titled as a research assistant

d_i = number of faculty needed for the exam i

w_k = "weighting value" of the faculty's preferences according to their title in the objective function k = {A,B,C,D}

Decision Variables

$$x_{ij} = \begin{cases} 1 \text{ faculty j is assigned to the exam i} \quad i = 1,..,N;\ j = 1,..,M \\ 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{otherwise} \end{cases}$$

Model

$$\max Z=\sum_{j=1}^{M} w_k \left[\sum_{i=1}^{N} c_{ij} \times x_{ij}\right] k=\{A,B,C,D\} \qquad (1)$$

Subject to the constraints;

$$\sum_{j=1}^{M} x_{ij} = d_i \quad i = 1,2 \ldots., N \quad (2)$$

$$A_1 \leq \sum_{i=1}^{N} x_{ij} \leq A_2 \quad j = 1,..,H \quad (3)$$

$$B_1 \leq \sum_{i=1}^{N} x_{ij} \leq B_2 \quad j = H+1, \ldots, S \quad (4)$$

$$C_1 \leq N \sum_{i=1}^{m} x_{ij} \leq C_2 \quad j = S+1, \ldots, T \quad (5)$$

$$D_1 \leq N \sum_{i=1}^{m} x_{ij} \leq D_2 \quad j = T+1, \ldots, M \quad (6)$$

$$\sum_{i=1}^{N} x_{ij} \in \{0,1\} \quad j = 1,2 \ldots., T \quad (7)$$

w_k : the weighting value for the preferences of the faculties. k={A,B,C,D}

$w_D = 10$ $w_D$starts with the value '10'

w_C = (s(D)+1) x w_D

w_B = (s(C)+1) x w_C

w_A = (s(B)+1) x w_B

The objective function, the equation 1, is aimed to make assignment to all the exams with maximum value. Equation 2 is to ensure that the number of faculties assigned is equal to the number of faculty required for each exam. Equations 3, 4, 5 and 6 are to ensure the number of assignments for each faculty with the same title is between limitations defined before. Finally equation 7 is the binary constraint.

IV. NUMERICLE EXAMPLE

The assignment problem is illustrated on a numerical example that is developed from a real case situation. It is developed from the assignment of the faculties to the exams. There are 38 exams. The number of faculty needed for each exam depends on the number of students having the exam so all the exams have different number of need of faculty. The number of faculty needed for each exam is defined by "di".

IJARE

There are 15 faculties that are going to be assigned to the exams. The number of the faculties titled as associate professor is 2, assistant professor is 8, PhD research assistant is 1 and as research assistant is 4. The entire faculties express their preference of the specific time for the exam on a sheet by giving coefficient to the desired time of having task. c_{ij} value is `10` to express to willingness to be assigned to the exams, '-99' for not likely to be assigned to the exam and '-10' for the time that they rather not to have an exam. According to their title, faculties' can be assigned to a different number of exams. Also the importance given to the preferences of the faculties is differing according to the title of the faculties. It is called as "importance value".

The required number of faculty for each exam is given as follows;

$d_1, d_2, \ldots\ldots, d_{38}$ = 7, 5, 7, 5, 5, 3, 9, 5, 5, 5, 3, 7, 5, 5, 5, 5, 3, 5, 7, 7, 7, 3, 7, 5, 3, 7, 3, 5, 7, 5, 9, 3, 5, 5, 3, 7, 7, 7.

The limitations for each title is given as;

$A_1 = 7$, $A_2 = 8$, $B_1 = 11$, $B_2 = 12$, $C_1 = 15$, $C_2 = 16$, $D_1 = 19$, $D_2 = 20$

Sets

A = {1,2} set of academicians titled as an associate professor indexed by j

B = {3,4,5,6,7,8,9,10} set of academicians titled as assistant professor indexed by j

C = {11} set of academician titled as a PhD research assistant indexed by j

D = {12,13,14,15} set of academicians titled as an research assistant indexed by j

k = {A,B,C,D} set of weights of the criteria's in the objective functions indexed by k

N = {1,…, 38} set of exams indexed by i

Model Parameters

i = the number of the exams i = {1,…..,38}

j = the number of the faculty j = {1,…..,15}

c_{ij} = the wish value of the faculty j on the time of the exam i.

"w_k"is defined as the weighting value of the faculties preferences according to their title. Since there are only 4 types of title; k= {A, B, C, D}

w_D= 10 w_D starts with the value '10'

w_C= 50, since there are 4 research assistant w_C is calculated as (s(D)+1) x w_D = 50

w_B = 100, since there is 1 PhD research assistant w_B is calculated as (s(C)+1) x w_C = 100

w_A = 900, since there are 8 assistant professor w_A is calculated as (s(B)+1) x w_B = 900

The objective of this mathematical model is to provide to assign needed number of faculties for each exam by maximizing the faculty willingness. Weighted objective maximization function is listed as below;

$$\boldsymbol{maxZ}=\sum_{j=1}^{15} w_k \left[\sum_{i=1}^{38} c_{ij} \times x_{ij}\right] \text{k}=\{\text{A,B,C,D}\} \quad (1)$$

Decision Variables

$$x_{ij}=\begin{cases}1 & \text{if the faculty j is assigned to the exam i } \ i = 1,..,38;\ j = 1,..,15\\ 0 & \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{otherwise}\end{cases}$$

Subject to the Constraints:

$$\sum_{j=1}^{15} x_{ij} = d_i \quad i = 1,2\ldots.,38 (2)$$

$$7 \leq \sum_{i=1}^{38} x_{ij} \leq 8 \quad j = 1,2 (3)$$

$$11 \leq \sum_{i=1}^{38} x_{ij} \leq 12 \quad j = 3,\ldots,10 (4)$$

$$15 \leq \sum_{i=1}^{38} x_{ij} \leq 16 \quad j = 11 (5)$$

$$19 \leq \sum_{i=1}^{38} x_{ij} \leq 20 \quad j = 12,\ldots,15 (6)$$

$$\sum_{i=1}^{38} x_{ij} \in \{0,1\} \quad j = 1,2\ldots.,15 \quad (7)$$

The equation 2 is to assign the right number of faculties needed for each exam. The equations 3,4,5,6 are used for balancing the assignments of the faculties with the same title. With these constraints the number of assigned exams to each faculty has to be within given limitations according to their title and it is so close to the others with the same title. The equation 7 is binary constraint.

According to the desired scores came from the faculty, the values "10" is given for the desired time to be assigned to the exams. And the number "-99" is given for the time of the exam that is not wanted to be taken. Also if there is not a proofed the reason but the faculty does not want to take the task at the time of an exam we gave "-10" to that time.The values can be seen from the table.1.

IJARE

Table 1

	Associate prof					Assistant prof					Phd Res. Assist.		Res.Assist.		
faculty	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
exam	900	900	100	100	100	100	100	100	100	100	40	10	10	10	10
d1	-99	10	10	-99	-10	10	-99	-99	-99	10	-10	10	-10	10	10
d2	-99	10	10	10	10	10	-99	-99	10	10	-10	10	10	10	10
d3	-99	10	-99	10	10	10	-99	-99	10	10	-10	10	10	10	10
d4	-99	10	-99	10	10	10	-99	-99	10	10	-10	10	10	10	10
d5	-99	10	-99	10	10	10	-99	10	-10	10	-10	10	10	10	10
d6	-99	10	-99	-99	10	10	-99	10	-10	10	-10	10	-10	10	10
d7	10	-99	10	-99	-99	10	10	-10	-99	10	-10	10	10	10	10
d8	10	-99	10	10	-99	10	10	-10	-10	10	-10	10	10	10	10
d9	10	-99	10	10	-99	10	10	-10	-10	10	-10	10	10	10	10
d10	10	-99	10	10	-99	10	10	-10	-10	10	-10	10	10	10	10
d11	10	-99	10	10	-99	10	10	10	-10	10	-10	10	10	10	10
d12	-99	-99	-99	-99	-10	10	10	-10	-99	-10	-10	10	10	-10	-99
d13	-99	-99	-99	10	10	10	10	-10	-10	-10	-10	10	10	-10	-99
d14	-99	-99	-99	10	10	10	10	-10	-10	-10	-10	10	10	-10	-99
d15	-99	-99	-99	10	10	10	10	10	-10	-10	-10	10	10	-10	-99
d16	-99	-99	10	-99	-10	10	-99	10	-99	-10	-10	10	10	-99	10
d17	-99	-99	10	10	10	10	-99	10	-10	-10	-10	10	10	-99	10
d18	-99	-99	10	10	10	10	-99	10	-10	-10	-10	10	10	-99	10
d19	-99	-99	10	10	10	10	-99	10	-10	-10	-10	10	10	-99	10
d20	-99	-99	10	10	10	10	-99	10	-10	-10	-10	10	10	-99	10
d21	-99	10	-99	-99	-10	10	-99	-10	10	10	10	10	-10	-99	10
d22	-99	10	-99	10	-99	10	-99	-10	10	10	10	10	-10	-99	10
d23	-99	10	-99	10	-99	10	-99	-10	10	10	10	10	-10	-99	10
d24	-99	10	-99	10	-99	10	-99	-10	10	10	10	10	-10	-99	10
d25	-99	10	-99	10	-99	10	-99	10	10	10	10	10	-10	-99	10
d26	10	10	10	-99	-10	10	-99	-99	10	10	10	10	-10	10	10
d27	10	10	10	10	10	10	-99	-99	10	10	10	10	10	10	10
d28	10	10	10	10	10	10	-99	-99	10	10	10	10	10	10	10
d29	10	10	10	10	10	10	-99	-99	10	10	10	10	10	10	10
d30	10	10	10	10	10	10	-99	5	10	10	10	10	10	10	10
d31	10	-99	-99	-99	-99	10	10	-10	10	-10	10	-99	-10	10	10
d32	10	-99	-99	10	-99	10	10	-10	10	-10	10	-99	-10	10	10
d33	10	-99	-99	10	-99	10	10	-10	10	-10	10	-99	-10	10	10
d34	10	-99	-99	10	-99	10	10	-10	10	-10	10	-99	-10	10	10
d35	10	-99	-99	10	-99	10	10	10	10	-10	10	-99	-10	10	10
d36	10	-99	-99	-99	-99	10	10	-10	-10	-10	10	10	10	-10	-99
d37	10	-99	-99	10	10	10	10	-10	-10	-10	10	10	10	-10	-99
d38	10	-99	-99	10	10	10	10	-10	-10	-10	10	10	10	-10	-99

V. RESULTS AND CONCLUSİONS

The model is solved with the open solver program and the results are shown in the Table.2. As the model solved, it is seen that the model gave the most suitable assignment according to the faculty's preferences. In case of a bottleneck, model assigned the task to the faculty with the lower title. As it can be seen from the figure.1 there are 8 assignments to the first faculty. He/she is assigned to the exams numbered as 7, 26, 31, 33, 34, 36, 37, 38. 2nd faculty is assigned to 8 exams numbered as 1, 2, 3, 4, 21, 23, 24, 26.As I worked with the real case, it is seen that the faculties are not satisfied with the assignment if the number of the days having exam is more than they desired. Some of the faculties who do not have a constraint for the specific date and time but wanted to have the task not more than 3 or 4 days. The developed model can be changed by taking into account of the faculty's number of the days having exam. Also this problem can be solved by the heuristics for further studies.

PAPER

Table 2

	Associate prof		Assistant prof								Phd Res. Assist.	Res.Assist.			
faculty exam	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
d1	0	1	1	0	0	1	0	0	0	1	0	1	0	1	1
d2	0	1	1	0	0	0	0	0	1	0	0	0	1	0	1
d3	0	1	0	1	0	1	0	0	1	0	0	1	1	0	1
d4	0	1	0	0	1	0	0	0	1	1	0	0	1	0	0
d5	0	0	0	0	1	0	0	1	0	0	0	0	1	1	1
d6	0	0	0	0	0	0	0	1	0	1	0	0	0	1	0
d7	1	0	1	0	0	1	1	0	0	1	0	1	1	1	1
d8	0	0	1	1	0	0	0	0	0	1	0	1	0	1	0
d9	0	0	1	1	0	0	1	0	0	0	0	1	0	1	0
d10	0	0	1	1	0	1	1	0	0	0	0	0	0	1	0
d11	0	0	0	0	0	0	0	1	0	1	0	0	0	1	0
d12	0	0	0	0	0	1	1	0	0	0	1	1	1	1	1
d13	0	0	0	1	1	0	1	0	0	0	0	1	1	0	0
d14	0	0	0	1	1	0	1	0	0	0	0	1	1	0	0
d15	0	0	0	1	1	0	1	1	0	0	0	0	1	0	0
d16	0	0	1	0	0	0	0	1	0	0	0	1	1	0	1
d17	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0
d18	0	0	1	0	1	0	0	1	0	0	0	0	1	0	1
d19	0	0	1	0	1	1	0	1	0	0	0	1	1	0	1
d20	0	0	1	1	1	0	0	1	0	0	0	1	1	0	1
d21	0	1	0	0	0	1	0	0	1	1	1	1	0	0	1
d22	0	0	0	0	0	0	0	0	0	0	1	1	0	0	1
d23	0	1	0	1	0	1	0	0	1	0	1	1	0	0	1
d24	0	1	0	0	0	0	0	0	1	1	1	0	0	0	1
d25	0	0	0	0	0	0	0	1	0	1	1	0	0	0	0
d26	1	1	0	0	0	0	0	0	1	0	1	1	0	1	1
d27	0	0	0	0	0	0	0	0	1	1	0	0	0	1	0
d28	0	0	0	0	1	0	0	0	0	0	1	1	0	1	1
d29	0	0	1	0	1	0	0	0	1	1	1	0	0	1	1
d30	0	0	0	0	0	0	0	1	0	1	1	0	1	1	0
d31	1	0	0	0	0	1	1	0	1	0	1	1	1	1	1
d32	0	0	0	0	0	0	0	0	1	0	0	0	0	1	1
d33	1	0	0	0	0	1	0	0	1	0	1	0	0	0	1
d34	1	0	0	1	0	1	0	0	0	0	1	0	0	1	0
d35	0	0	0	0	0	0	1	1	0	0	0	0	0	1	0
d36	1	0	0	0	0	1	1	0	0	0	1	1	1	1	0
d37	1	0	0	1	1	0	1	0	0	0	1	1	1	0	0
d38	1	0	0	1	1	0	1	0	0	0	1	1	1	0	0

IJARE

REFERENCES

[1] Alexander, S. On the history of combinatorial optimization (till 1960). Handbooks in Operations Research and Management Science: Discrete Optimization, 2005, 12: 1.

[2] Cai, X., & Li, K. N., A genetic algorithm for scheduling staff of mixed skills under multi-criteria. European Journal of Operational Research, 2000. 125(2), 359-369.

[3] Drexl, A., &Nikulin, Y.,Multicriteria airport gate assignment and Pareto simulated annealing. IIE Transactions, 2008, 40(4), 385-397.

[4] Lee, S. M., &Schniederjans, M. J., A multicriteria assignment problem: A goal programming approach. Interfaces, 1983, 13(4), 75-81.

[5] Lin, C. M., & Gen, M., Multi-criteria human resource allocation for solving multistage combinatorial optimization problems using multiobjective hybrid genetic algorithm. Expert Systems with Applications, 2008, 34(4), 2480-2490.

[6] Odior, A., Charles-Owaba, O., &Oyawale, F., Determining Feasible Solutions of a Multicriteria Assignment Problem. Journal of Applied Sciences and Environmental Management, 2008, 14(1)

[7] Reza Ramezanian, Abdullah Ezzatpanah,,Modeling and solving multi-objective mixed-model assembly line balancing and worker assignment problem, Computers & Industrial Engineering, Volume 87, September 2015, Pages 74–80.

[8] Ross, G.E., Zoltners, A.A., Weighted Assignment Models and Their Application, Northwestern University, July 1, 1979, pp: 683 – 696.

[9] ROY, Rupsha; RATHORE, Rukmani. Method of Finding an Optimal Solution of an Assignment Problem. International Journal of Research in Engineering Technology and Management, 2014, 2.1: 1-4.

[10] Rigopoulos, G., &Karadimas, N. V.,Military staff assignment approach utilizing multicriteria analysis. In Proceedings of the 5th WSEAS international conference on Communications and information technology, World Scientific and Engineering Academy and Society (WSEAS),2011, July, pp. 107-110.

[11] Scarelli, A., &Narula, S. C., A multicriteria assignment problem. Journal of MultiCriteria Decision Analysis, 2002, 11(2), 65-74.

[12] SOFI, N. A. Mathematical Programming: A Brief Historical Sketch and its Applications. Mathematical Theory and Modeling, 2014, 4.12: 70-77.

[13] Taha, H A,Operations Research: An Introduction, Seventh Edition, Thomson Press Ltd, 2002, India. pp: 196 -203.

IJARE