Solution of Mixture Problem Prioritized Raw Materials Using Mixed Integer Linear Programming

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Abstract— Mixture problem is the problem of obtaining one or more products from by-products having different contents under the required constraints. Mixture problems should be organized in a way to optimize a given objective function. This is generally a cost optimization, fulfilling specific constraints and conditions and this kind of problems mostly solved with linear programming models. Decision makers want to have the solutions for mixture problems under the different scenarios. One of these scenarios is to use the raw materials which have different priorities in the solution. In this study, a new mixed integer linear programming (MILP) model which finds a solution according to high priorities is developed. The priorities are determined according to raw materials availabilities between 1 and 4. The developed model is practiced on a feed mix problem as a mixture problem and the model is coded by object-oriented programming language C#.

Index Terms— **Mixing Problem, Linear Programming, Feed Mixing, Optimisation Application, Cost Optimisation**

I. INTRODUCTION

Feed mixture problem is mixing raw materials which have different features of feed formulation in accordance with the nutrient requirements, animal characteristics. The mixture is made according to the type of animal, age and breeding purposes and it has two important features including quality and cost for producer/decision maker. Usually linear solution techniques are used to solve problems of feed mixture.

First study about mathematical analysis of feed mixture was conducted by Frederic V. Waugh in 1951 using Linear Programming (LP) [1]. Later, depending on the type and feature of animals, a number of researchers have used the LP in order to optimize the cost of the feed mixture [2-6]. Also different methods such as Goal Programming (GP) [7, 8], Multi Goal Programming (MGP) [9, 10], Multi Objective Programming (MOP) [11-13] are used.

When the previous studies were examined, it was seen that Mixed Integer Linear Programming (MILP) method had never been used in the solution of feed mix problems. MILP problems are the problems in which some of the variables are restricted to be integers. While integers may have values in a specific interval, they may also have such values as 0-1 (true/false). Calculation of integer problems is difficult and complicated due to their combinational structure. As for MILP problems, they include variables having both integer values and continuous values. Thus, the calculation of MILP problems is more complicated than that of integer problems. Problems of this type belong to the Non-deterministic Polynomial-time (NP) class[14].

When the studies on feed formulation were examined, it has not been seen that selection of raw materials were made or modeled by giving priority.

While feed mixtures were being prepared, ability of decision maker to see the feed formulations with a broad perspective and produce solutions in different situations have strategic importance for the companies.

According to the state of raw materials in feed mills, it is sometimes requested to be included in the mixture as a higher priority than other raw materials. The reasons can be defined as;

- Abundance of available raw materials,
- Logistical strain of needed raw materials
- Supply rate of usable needed materials,
- The importance of contribution of material for the desired formulation.

In this study, decision maker identifies a priority between 1 and 4 to available raw materials. 1 shows that, the raw material has a high priority and 4 shows the raw material has low priority. These priorities can be changed at any time by decision makers. Raw materials which have high priority in feed formulation are forced into the mixture primarily. Feed formulation is done under the required constraints with priorities of decision maker.

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II. Material and Methods

A. Materials

The database used in this study was obtained from Selçuk University, Faculty of Veterinary Medicine, Department of Zootechnics and Husbandry[15]. The database was updated based on feed ingredients and price information. The database was set up according to mixed feed standards in Turkey[16].

The feed mix problem was tried to be solved based on different animal groups in the study. Concentrate feed mixes given to the sheep, cattle and rabbit groups in their growth periods. These animals are not classified into a certain group unlike poultry, thus their nourishment needs differs. Crude protein (CP), metabolizable energy (ME), calcium (Ca), phosphor (P), sodium (Na), crude fiber (CF) and crude ash (CA) are ingredient constraints in concentrate feed mix for these animals. Data in Table 1 is example of feeds to be used in feed mix. In this table, more than one feed and their contents can be seen. Table 2 is given to show specific constraints of feed mixes for desired animal species. These nutrients and corresponding values can be flexibly altered as long as vital needs of animals are sustained at certain levels.

TABLE I. The Nutrients Contents

		The nutrients contents										
		CP, %	*ME, Mcal /kg*	*Ca, %*	*P, %*	*Na, %*	*CF, %*	*Ash, %*	*DM, %*	*Cost ,Krş*	*Min*	*Max*
Nutrients	*Barley*	13,5	3,11	0,05	0,38	0,02	88	2,9	88	30	0	60
	Corn, 2950	10,1	2,95	0,02	0,29	0,02	88	1,5	88	25	0	70
	Cottonseed meal, solvent extracted, 26% CP	29,0	2,2	0,20	1,00	0,05	90	7,3	90	25	0	30
	Soybean meal, 44% CP	46,8	3	0,32	0,71	0,04	91	6,6	91	50	0	40
	Alfalfa meal, %15	17,3	2,06	1,37	0,24	0,08	90	9,9	90	30	0	20
	Dicalcium phosphate	0	0	24	18	0,03	100	100	100	55	0	3
	Limestone	0	0	35	0	0,06	100	100	100	2,5	0	7
	Sodium chloride	0	0	0	0	39,34	100	100	100	10	0,25	1
	Vitamin-mineral premix	0	0	0	0	0	100	0	100	250	0,25	0,35

TABLE II. The Nutritional Requirements In Rabbit Groups (constraints)

Animal species	Rabbit	
Animal type	Rabbit grower compound feed	
Nutrients	**Min**	**Max**
Crude protein [CP (%	16	-
Metabolizable energy [ME (kcal/kg)]	2,3	-
Calcium [Ca (%)]	0,8	1,4
Phosphorus [P (%)]	0,6	-
Sodium [Na (%)]	0,3	0,4
Crude fiber [CF (%)]	-	10
Ash [Ash (%)]	-	10

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B. Methods

a. Basic Linear Programming Model For Feed Mixing

Mixture problem is the problem of obtaining one or more products from by-products having different contents under the required constraints. Feed mix problem is also a kind of mixture problem and it aims to find the feed mix having the required properties by mixing the raw materials with different contents at specific ratios. An ideal feed mix should have the intended quality properties and should be obtained at minimum cost. Thus, feed mix problems should be organized in a way to optimize a given objective function. This is generally a cost optimization fulfilling specific constraints and conditions[17].

Linear Programming model for feed mixing has given as below;

Objective Function :

$$MinZ = \sum_{j=1}^{n} c_j x_j \quad (1)$$

Subject to:

$$p_i \leq \sum_{j=1}^{n} a_{ij} x_j \leq P_i, i = 1, \dots, m, \quad (2)$$

$$m_j \leq x_j \leq M_j, j = 1, \dots, n \quad (3)$$

$$\sum_{j=1}^{n} x_j = tq \quad (4)$$

In the basic feed mix model, j={1,2,...,n} displays the feeds in the mix while i={1,2,...,m} represents the ingredients of the raw materials in the mix. In the objective function, x_j is the amount of the raw material j in the mix and c_j refers to the cost of the raw material j in the unit amount. The objective function is the sum obtained through the multiplication of the amounts and unit costs of the raw materials included in the mix. As it is seen in Equation 1, the objective is cost minimization. In Equation 2, a_{ij} refers to the amount of the nutrient i in the raw material j while p_i refers to the minimum amount required for the nutrient i and P_i is the maximum amount required for the nutrient i. Depending upon the feed formulation selected through Equation 2, the contents of the raw materials to be included in the mix are restricted. In Equation 3, m_j is the minimum amount that the raw material j can be added into the mix while M_j refers to the maximum amount that the raw material j can be added into the mix. Equation 3 is used to keep the amounts of the raw materials to be used in the mix at the desired interval. In Equation 4, tq is employed so as to determine the amount of the total feed mix. Since feed mixes are calculated by percentage, the value of tq is considered to be 100.

b. New Mixed Integer Linear Programming Model For Nutrient Priority

In this study, the following new constraints and decision variables were added into the basic feed mixture model with the aim of producing alternative solutions for different animal species (sheep, cattle and rabbits).

Objective Function :

$$MinZ = \sum_{j=1}^{n} c_j x_j + 10^2 * \theta_1 + 10^3 * \theta_2 + 10^4 * \theta_3 + 10^5 * \theta_4 \quad (5)$$

In classic feed formulation model, objective function is cost minimization. New decision variables are added to the objective function of the new model. Equation 5 represents $\theta_1, \theta_2, \theta_3, \theta_4$ priority groups and has a binary form (0.1). If a feed which belongs to any priority group is selected, the decision variable of the relevant group becomes "1". The coefficient which belongs to each priority groups is increased linearly according to the importance of the group. While the coefficients of the first priority feed group is 100, the second group has been determined as 1000. Ingredients in the feed mixture are prioritized according to the group priorities.

Subject to:

$$p_i \leq \sum_{j=1}^{n} a_{ij} x_j \leq P_i, i = 1, \dots, m, \quad (6)$$

$$m_j \leq x_j \leq M_j, j = 1, \dots, n \quad (7)$$

$$\sum_{j=1}^{n} x_j = tq \quad (8)$$

$$x_j \leq U_j * M, j = 1, \dots, n \quad (9)$$

$$\theta_t \geq \theta_{t+1}, t = 1,2,3,4 \quad (10)$$

$$\theta_j \geq U_j * g_{tj}, j = 1, \dots, n \quad (11)$$

Equation 6, Equation 7 and Equation 8 is same of the Equation 2, Equation 3 and Equation 4 respectively. In this study, new constraints added to the new model. In the Equation 9, U_j is a binary-type variable while M refers to a large number. In this study, M was accepted to be 1000. Equation 9 is the constraint which helps to determine whether a raw material j is added into the mix. In the Equation 10, θ_t is a binary-type variable and t is a group number t={1,2,3,4}. Equation 10 is the constraint prevents selection of high priority groups. In the Equation 11, g_{tj} is the binary type variable which shows the group of the raw materials. Equation 11 is the constraint which helps determination whether a group t is added into the mix.

III. Feed Formulation Software

In this study, developed MILP model has been implemented in feed formulations. In the present study, fully object-oriented programming language .Net technology was used while Solver.Foundation was employed in order to carry out the solving processes (Figure 1)

Decision-makers can find feed formulation with this software by selecting the raw materials in the database within constraints which is determined. Also alternative solutions are presented according to the priorities of the feed using models developed for decision makers.

In Part 1 in the software, menus are available which decision makers will use.

Using this menu, decision-makers can select, modify and delete the raw materials which they want to use. Besides, they can choose which feed formulation mixture will be done. In order to be made feed formulations manually, it can calculate according to the amount of raw material mixture which has entered. Second section of software provides information about the selected feed formulation. Third section provides coloring codes belonging to priority groups. Fourth section contains information about the amount of costs and mix for classic feed formulations. Fifth section provides information about the quantity and mix for improved feed formulation costs. Sixth section lists selected raw materials for making feed formulation by decision makers. Seventh section provides information about the lower and upper limits of feed formulations of which mixture will be done. Besides this, as a result of LP and MILP models, amount of each ingredient is given.

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In eighth section feed formulation which is prepared using conventional mixture model is given. In ninth section feed formulation mixture prepared by using a newly developed model is given. Thus, decision makers can make better decisions.

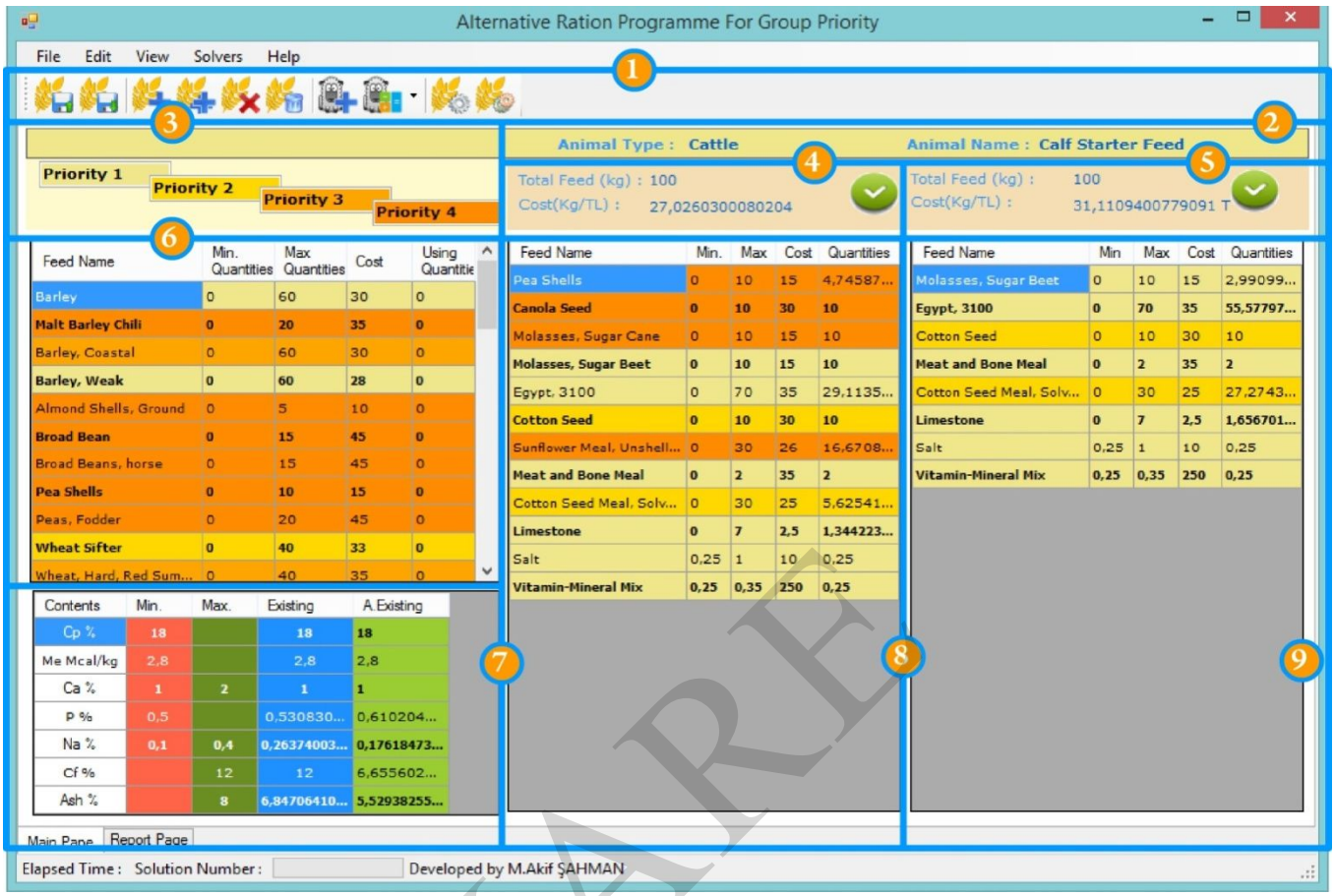


Fig. **1** :Alternate Feed Mix Software Interface

IV. Computational Experiment

In this study, sheep, cattle and rabbit feed formulations were determined on experimental groups of animals. Lamb Starting Feed, Cattle - Calf Starter Feed and Rabbit - Rabbit Growing Feed are selected feed formulations.

TABLE III. Comparison of Classic Mixing Model (LP) and New Improved Model (MILP)

Feed Name	Model	Priority Groups (Used Raw Material Number)				Cost (TL)
		1	2	3	4	
Sheep-Lamb Starting Feed	LP	6	2	0	4	26,97
Sheep-Lamb Starting Feed	**MILP**	**6**	**2**	**0**	**0**	**30,91**
Cattle-Calf Starter Feed	LP	6	2	0	4	27,03
Cattle-Calf Starter Feed	**MILP**	**6**	**2**	**0**	**0**	**31,11**
Rabbit-Rabbit Growing Feed	LP	7	2	1	3	22,11
Rabbit-Rabbit Growing Feed	**MILP**	**7**	**2**	**0**	**0**	**24,01**

When feed formulation which is called Sheep-Lamb Starting Feed is solved for both LP model and developed MILP model, it is seen LP use raw materials which belong to lowest priority group although its cost value is lower. MILP model can analyze the feed formulations which have only 1st and 2nd priority raw materials. It is observed that the above mentioned conditions also valid for other feed formulations (Table 3).

From Figure 2, when a feed formulation called Rabbit-Rabbit Feed Growing resolved for the newly developed MILP model, it is seen that the first and second primary raw materials have been used.

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Total Feed (kg) : 100
Cost(Kg/TL) : 24,0199368406941 TL

Feed Name	Min	Max	Cost	Quantities
Barley	0	60	30	41,2740...
Molasses, Sugar Beet	**0**	**10**	**15**	**10**
Now the Cotton Ginning	0	15	9	13,1379...
Meat and Bone Meal	**0**	**2**	**35**	**2**
Cotton Seed Meal, Solv...	0	30	25	30
Dicalcium Phosphate (D...	**0**	**3**	**55**	**0,026201...**
Limestone	0	7	2,5	2,87056...
Salt	**0,25**	**1**	**10**	**0,441208...**
Vitamin-Mineral Mix	0,25	0,35	250	0,25

Fig. **2**: Rabbit-Rabbit Growing Feed Formulation MILP (Barley is in the First Priority Group)

It is observed that most of the raw material entering to "Rabbit-Rabbit Growing Feed" is Barley. Barley raw material is seen as a member of the group which has first priority. When Barley raw material is included in the second priority group, solution can be seen from following Figure 3.

Total Feed (kg) : 100
Cost(Kg/TL) : 24,0199368406941 TL

Feed Name	Min	Max	Cost	Quantities
Barley	0	60	30	41,2740...
Molasses, Sugar Beet	**0**	**10**	**15**	**10**
Now the Cotton Ginning	0	15	9	13,1379...
Meat and Bone Meal	**0**	**2**	**35**	**2**
Cotton Seed Meal, Solv...	0	30	25	30
Dicalcium Phosphate (D...	**0**	**3**	**55**	**0,026201...**
Limestone	0	7	2,5	2,87056...
Salt	**0,25**	**1**	**10**	**0,441208...**
Vitamin-Mineral Mix	0,25	0,35	250	0,25

Fig. **3** : Rabbit-Rabbit Growing Feed Formulation MILP (Barley is in the Second Priority Group)

In Figure 3, it is seen that Barley has been used again because of the reason that even the second priority raw materials run out. This cannot change the objective function value. But how the solution changes when Barley raw material is included in the raw material group which has third priority, can be seen from Figure 4.

Total Feed (kg) : 100
Cost(Kg/TL) : 24,9580636717832 TL

Feed Name	Min	Max	Cost	Quantities
Barley, Weak	0	60	28	50,9223...
Molasses, Sugar Beet	**0**	**10**	**15**	**10**
Now the Cotton Ginning	0	15	9	3,25097...
Meat and Bone Meal	**0**	**2**	**35**	**1,838281...**
Cotton Seed Meal, Solv...	0	30	25	30
Limestone	**0**	**7**	**2,5**	**3,133485...**
Salt	0,25	1	10	0,60495...
Vitamin-Mineral Mix	**0,25**	**0,35**	**250**	**0,25**

Fig. **4** : Rabbit-Rabbit Growing Feed Formulation MILP (Barley is in the Third Priority Group)

From Figure 4 it can be seen that "Barley" has been used no longer and instead of it, raw material which is called "Barley Weak" which has second priority are being used. When "Barley" is the fourth member of the group of raw materials, the results are as Figure 4 and do not change.

Under these results, according to the priorities of the raw material improved MILP model is incorporated into the feed formulation is clearly seen.

V. Conclusion and Future Works

In mixture problems, decision makers use available raw material within the frame of required constraint to obtain the desired product. While obtaining the product, mixture has to comply with the constraints and have to be low cost. Decision makers usually challenged by availability and abundance of a raw material, logistical strain and supply rate for needed material and individual contribution of any material in this kind of mixture problems. Thus, a MILP mixture model has been proposed to solve such situations. In the developed model, if the first priority is not able to provide constraints of mixture, the second priority is being used. If the second primary raw materials are not also sufficient, the third and fourth priority raw materials are used. In developed MILP model, the mixture is intended to minimalize the costs of raw materials and decision variables belong to using priority groups. Thus, in objective function there are both costs and decision variables belong to using of the priority groups.

In this study, MILP model which is proposed for mixture problem was applied at feed formulation which is one of the types of model mixture problem. Considering the trials in the

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MILP model, high-priority materials are selected by the decision-makers, at the same time costs minimization and optimizations are done. So decision-makers can create a scenario what is wanted and it can be managed.

In the future studies, administrative models can be developed to be able to manage different stores and businesses. For models, which solution duration and complexity is higher, heuristic-based solutions can be proposed.

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