Numerical Analysis of Transient Response in a Single-Pass Cross flow Heat Exchanger

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***Abstract*— Crossflow heat exchangers are commonly employed in process critical applications in industries. A heat exchanger can be subjected to flow or temperature perturbation during its regular operation. It should be noted that a heat exchanger would require a finite time to overcome such perturbations. In this study, the transient response of a single-pass crossflow heat exchanger with variable inlet temperatures and mass flow rates was determined. Herein, the analysis of the transient performance of cross flow heat exchangers is conducted on a dimensionless basis. In every instance, the energy balance equations were solved using an explicit finite difference method. Numerical predictions were obtained for cases where both the fluids were subjected to step changes in inlet temperature, coupled with step mass flow rate change of the fluids. The analysis can be readily extended to study different flow circuiting and inlet conditions.**

***Index Terms*— Cross Flow Heat Exchangers, Transient Response**

I. INTRODUCTION

The design of a heat exchanger presumes accurate knowledge of fluid inlet temperatures and flow rates. However, in many instances a heat exchanger does not operate at the design point. Under those circumstances, it is necessary to predict heat exchanger performance at off-design conditions. Heat exchangers do not instantaneously respond to changes in the inlet fluid temperatures and mass flow rates, and a finite time is required for the heat exchanger to stabilize from such variations. Perturbations in inlet fluid temperatures and flow rates will cause disturbances in the response of the heat exchanger, and these may introduce undesirable consequences to the process system. This paper addresses the development of a transient finite difference sensible performance model for a single pass crossflow heat exchanger. Variations of the inlet fluid temperatures and mass flow rates as a function of time are considered. The paper details the pertinent governing equations, and presents the results for various cases. There are a great number of references available in the literature pertaining to transient heat exchanger performance modeling. Only the most pertinent references are discussed herein. Spiga and Spiga [1] studied the two-dimensional transient behavior of gas-to-gas crossflow heat exchangers by solving the thermal balance equation by analytical methods. Spiga and Spiga [2] provided non-dimensional solutions for transient temperature analysis of direct transfer crossflow heat exchangers. Spiga and Spiga [3] studied the step response of a single-pass crossflow heat exchanger with finite wall capacitance. Mishra et al. [4] numerically investigated the transient behavior of crossflow heat exchangers with longitudinal conduction and axial dispersion for step, ramp, and exponential hot fluid inlet temperature perturbations. Mishra et al. [5] developed a numerical scheme for studying the transient behavior of crossflow heat exchangers having finite wall capacitance for perturbations in both temperature and flow. The explicit finite difference approach was used, and the results were presented for step, ramp, exponential, and sinusoidal variation in the hot fluid inlet temperature. Dwivedi and Das [6] presented a transient model to analyze the transient response of plate heat exchangers subjected to a step flow variation. Mishra et al. [7] analyzed the effects of temperature and flow non-uniformities on the transient response of crossflow heat exchangers. Silaipillayarputhur and Idem [8] studied the step response of a single-pass crossflow heat exchanger with variable inlet conditions. In every instance the energy balance equations were solved using an implicit central finite difference method. Performance calculations were conducted for heat exchangers operating initially at steady state, where step flowrate changes of the minimum and maximum capacity rate fluids were imposed in the absence of any temperature perturbations. Likewise numerical predictions were obtained for cases where both the minimum or maximum capacity rate fluids were subjected to step changes in inlet temperature, coupled with step mass flowrate changes of the fluids. Silaipillayarputhur and Idem [9] addressed the development of a transient sensible performance model for a multi-pass counter crossflow heat exchanger where the fluids were unmixed in a pass and were mixed between the passes. Variations of the inlet fluid temperatures and mass flowrates as a function of time were considered. To validate the transient performance model, the results obtained from the transient performance model for large times were compared with predictions from a steady performance model. Silaipillayarputhur and Idem [10] addressed the step response of a multi-pass crossflow heat

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exchanger for variable inlet temperatures and mass flowrates. In every instance overall parallel and counterflow circuiting was considered. The energy balance equations were solved using an implicit central finite difference method. Numerical predictions were obtained for the case where the minimum capacity rate fluid was subjected to a step change in inlet temperature while mass flowrates for both fluids were maintained constant. Under those circumstances it was demonstrated that for short times the transient response of a counterflow heat exchanger was nearly independent of the number of passes. Likewise the transient performance was calculated when a step mass flowrate change of the minimum capacity rate fluid was imposed, in the absence of any temperature perturbations. In that situation the transient heat exchanger response was influenced by the number of passes for both parallel and counterflow circuiting.

II. NOMENCLATURE

A - Heat transfer surface area
c - Specific heat of fluid
C_r - Capacity rate ratio
E - Inverse of heat capacity rate ratio
h - Heat transfer coefficient
$\dot{m}$ - Mass flow rate
M - Grid size in 'a' fluid direction
n - Number of grids chosen for the numerical integration
N - Grid size in 'b' fluid direction
N - Dimensionless heat exchanger length
NTU - Overall number of transfer units
R - Heat transfer resistance ratio
T - Dimensionless temperature
$\overline{T}$ - Average outlet temperature
V - Heat capacitance ratio
x - Dimensionless heat exchanger coordinate
y - Dimensionless heat exchanger coordinate
β - Constant = 0.8
γ - Ratio of mass flow rate at any time t to that at initial time level
θ - Temperature
$\Delta\theta$ - Temperature change of stepped fluid
ϕ - Perturbation in the fluid inlet temperature
ϕ_1 - Perturbation in the fluid mass flow rate
Δx - Grid spacing
Δy - Grid spacing
a - Primary fluid
b - Secondary fluid
w - Heat exchanger wall
' - Performance parameter at any time t after a flow rate perturbation

III. TRANSIENT PERFORMANCE MODEL

The current study directly follows from the work presented in [5, 8, 9, 10]. Figure 1 depicts a schematic representation of the single-pass crossflow heat exchanger considered in this study. Fluid 'a' and fluid 'b' exchange heat through a separating solid wall, and the flow direction of both fluids is normal to one another. In general, the designation of the 'a' fluid or 'b' fluid is completely arbitrary. However in this paper, it is convenient to define the 'a' fluid such that it has a capacity rate less than or equal to that of the 'b' fluid. It is proposed that either fluid can experience a change in inlet temperature and/or mass flow rate. In order to consider a manageable number of cases, the present paper is restricted to situations where one fluid undergoes changes in inlet fluid conditions while the inlet conditions of the other fluid are presumed to remain constant in time. That constraint need not be imposed, and [8] considers a number 3 of other instances where transient inlet conditions of both fluids vary simultaneously. The assumptions made in this study, consistent with those outlined in [8, 9, 10], are as follows:

(i) the thermo-physical properties of both the fluids and walls are constant and uniform, (ii) both fluids are unmixed in the given pass, (iii) there is no phase change in the fluids, (iv) there is no internal heat generation in either fluid, (v) the heat exchanger does not exchange heat with the surroundings, (vi) the temperature of both fluids does not vary in a direction normal to the separating solid wall, indicating that the fluid and wall temperature variations are two-dimensional, (vii) the film heat transfer coefficient is solely a function of fluid velocity, and changes with time as the flow rate simultaneously changes, and (viii) the effects of fouling resistances are negligible.

The capacity rate ratio can be expressed as

$$C_r = \frac{(\dot{m}c)_a}{(\dot{m}c)_b} = \frac{1}{E} \tag{1}$$

The heat transfer resistance ratio is defined in terms of the convection heat transfer environments on either side of the exchanger wall, such that [9]

$$R = \frac{(hA)_b}{(hA)_a} \tag{2}$$

Per [5] the flow perturbation can be expressed by defining the ratio of mass flow rate at any time t to that at initial time level such that

$$\gamma_a = \frac{\dot{m}'_a}{\dot{m}_a}; \gamma_b = \frac{\dot{m}'_b}{\dot{m}'_b} \tag{3}$$

Assuming fully developed turbulent flow, the heat transfer coefficient is proportional to mass flow rate. Therefore, the heat transfer coefficient at any time t is defined as [5]

$$h'_a = \gamma_a^{\beta} h_a \tag{4}$$

and: $h'_b = \gamma_b^{\beta} h_b$ (5)

where the constant β is assumed to be 0.8. Applying the above assumptions, the conservation of energy for both the fluids and for the solid wall can be expressed as follows [5]:

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$$\gamma_a \frac{\partial T_a}{\partial x} + V_a \frac{\partial T_a}{\partial t} + \gamma_a^{\beta} T_a = \gamma_a^{\beta} T_w \tag{6}$$

$$\gamma_b \frac{\partial T_b}{\partial y} + \frac{V_b}{R} \frac{\partial T_b}{\partial t} + \gamma_b^{\beta} T_b = \gamma_b^{\beta} T_w \tag{7}$$

$$\frac{\partial T_w}{\partial t} + (\gamma_a^{\beta} + R\gamma_b^{\beta}) T_w = \gamma_a^{\beta} T_a + R\gamma_b^{\beta} T_b \tag{8}$$

The quantities V_a and V_b are the capacitance ratios for both the fluids; complete details are provided in [8]. Let θ represent the fluid or heat exchanger wall temperature at any given instant and location. In that case, the dimensionless temperatures are defined such that [2]

$$T_j = \frac{\theta_j - \theta_{j,initial}}{\Delta\theta} \; ; \; j = a,b,w \tag{9}$$

The initial number of transfer units can be expressed as [2]

$$\frac{1}{NTU} = (\dot{m}c)_{min} \left(\frac{1}{(hA)_a} + \frac{1}{(hA)_b} \right) \tag{10}$$

The number of transfer units at any time t is given as [2]

$$\frac{1}{NTU'} = (\dot{m}c)_{min} \left(\frac{1}{(h'A)_a} + \frac{1}{(h'A)_b} \right) \tag{11}$$

Upon rearranging the above equation yields

$$\frac{1}{NTU'} = \frac{\gamma_a R}{NTU(R+1)\gamma_a{}^{\beta}} + \frac{\gamma_a}{NTU(R+1)\gamma_b{}^{\beta}} \tag{12}$$

From [5], after an inlet fluid mass flow rate change has occurred the heat exchanger dimensionless length quantities for $(\dot{m}c)_a = (\dot{m}c)_{min}$, i.e., $\frac{\gamma_b}{\gamma_a} \cdot \frac{1}{C_r} > 1$, are given by

$$N_a = NTU'\gamma_a \left(\frac{1}{\gamma_a{}^{\beta}} + \frac{1}{R\gamma_b{}^{\beta}} \right) \tag{13}$$

and:

$$N_b = \frac{NTU'\gamma_a}{r} \left(\frac{R}{\gamma_a{}^{\beta}} + \frac{1}{\gamma_b{}^{\beta}} \right) \tag{14}$$

Likewise for $(\dot{m}c)_b = (\dot{m}c)_{min}$, i.e., $\frac{\gamma_b}{\gamma_a} \cdot \frac{1}{C_r} < 1$

$$N_a = NTU'\gamma_b r \left(\frac{1}{\gamma_a{}^{\beta}} + \frac{1}{R\gamma_b{}^{\beta}} \right) \tag{15}$$

and:

$$N_b = NTU'\gamma_b \left(\frac{R}{\gamma_a^{\beta}} + \frac{1}{\gamma_b^{\beta}} \right) \tag{16}$$

At time t = 0, the heat exchanger is assumed to be operating at steady state and thus the dimensionless temperatures are given as

$$T_a(x,y,0) = T_b(x,y,0) = T_w(x,y,0) = 0 \tag{17}$$

Initially, both the fluids have a steady mass flow rate. Therefore at time t = 0

$$\gamma_a = \gamma_b = 1 \tag{18}$$

If $\phi_1(t)$ is the perturbation in the inlet mass flow rate of either fluid, then at any time t > 0, for a step change in the fluid flow rate

$$\gamma_a = \phi_{1,a} \; ; \; \gamma_b = \phi_{1,b} \tag{19}$$

The mass flow rate disturbance functions $\phi_{1,a}$ and $\phi_{1,b}$ are arbitrary, and generally can be different for either fluid. If $\phi(t)$ is the perturbation in the 'a' fluid inlet temperature, then at any time t > 0, for a step change in temperature

$$T_a(0,y,t) = \phi = 1 \tag{20}$$

Fluid 'b' enters the heat exchanger at a known constant temperature, hence at any time t > 0,

$$T_b(x,0,t) = 0 \tag{21}$$

The boundary and initial conditions as expressed by Equations (17) through (21) will yield steady heat exchanger performance in the limit as $t \to \infty$.

Equations (6) through (8) are solved using an explicit finite difference method. The time step size for the transient analysis is chosen as $\Delta t = 0.05$, and the grid spacing for both the transient and steady analysis is chosen as $M \times N = 100 \times 100$. Under these circumstances the model always exhibits grid independence; refer to [8].

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IV. RESULTS

Based on the concepts and the equations described in the previous sections, a model is developed to study and validate the transient finite difference sensible single-pass heat exchanger performance. The transient performance of the heat exchanger is studied over a range of parameters, and the salient observations pertaining to each case are discussed below. Figure 2 depicts the mean fluid exit temperatures from the transient performance model plotted as a function of dimensionless time due to a step temperature change of the 'a' fluid inlet temperature at different NTU values, where E = R = 1, V = 0, $\gamma a = \gamma b = 1$. In this case, it is assumed that both the fluids have a steady mass flow rate. The results from the transient performance model are compared with the analytical results presented in [1]. The fluid temperatures from [1] are presented as discrete points and the temperatures determined by the transient finite difference model are shown as solid lines. Figure 2 depicts excellent agreement between the results obtained from the current finite difference approach and the results obtained through analytical approach [1], thus promoting confidence in the transient finite difference performance model. Figures 3 and 4 depict the mean fluid exit temperatures calculated from the transient performance model plotted as a function of dimensionless time, due to a step change of the 'a' fluid inlet temperature coupled with a step flow rate change of the 'a' fluid. These cases pertain to the situation where E = R = V = NTU = 1 and $\gamma b = 1$, prior to the imposition of changes of inlet fluid conditions. In every instance, the transient response of the 'a' fluid displays a time lag, where outlet fluid temperature changes are not initially apparent. The time lag becomes progressively shorter as the step change flow rate ratio increases. Referring to Figure 3, for $\gamma a \geq 2$ the temperature response of the 'a' fluid is essentially instantaneous. In contrast, the temperature response of the 'b' fluid is always instantaneous. This is because for the outlet temperature of the stepped fluid to exhibit a noticeable change, it is necessary for the temperature disturbance in the 'a' fluid to propagate throughout the length of the exchanger. The outlet temperature of the 'b' fluid changes immediately, because that fluid adjoins the stepped fluid at the heat exchanger outlet. As the flow perturbation of the 'a' fluid is increased, the steady exit temperature of the 'a' fluid increases. Table 1 presents the steady state dimensionless temperatures from the transient performance model.

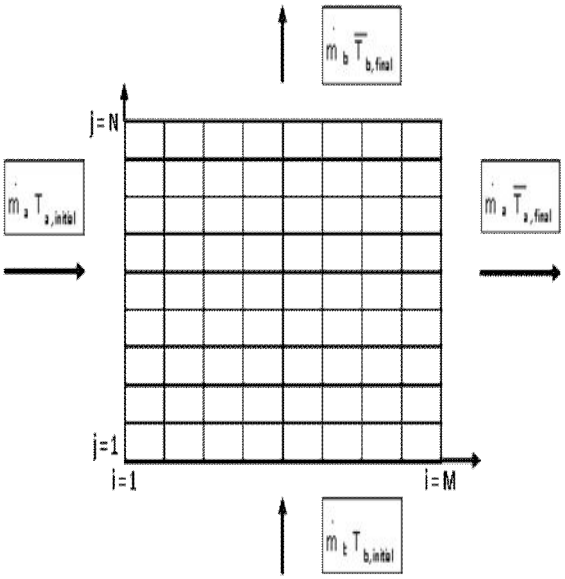


Figure 1 Finite Difference Grid of a Cross Flow Heat Exchanger

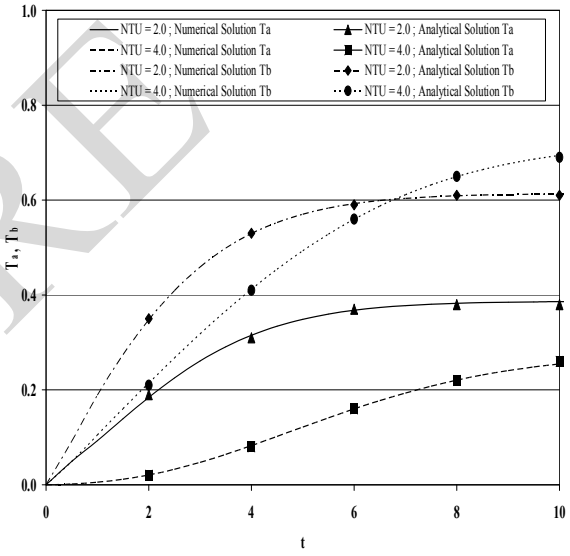


Fig.1. Mean fluid exit temperature of the hot and cold fluids due to a step temperature change of the 'a' fluid inlet temperature; E = R = 1, V = 0, $\gamma a = \gamma b = 1$

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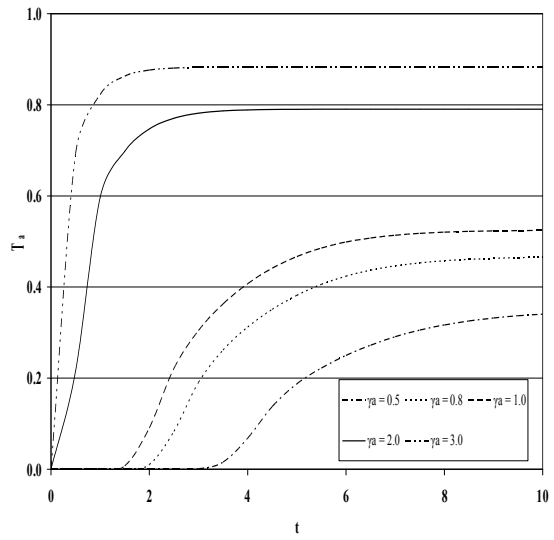


Figure 3. Mean fluid exit temperature of the 'a' fluid due to a step change of the hot fluid inlet temperature coupled with a step flow rate change of the 'a' fluid; initially E = R = V = NTU = 1; γb = 1

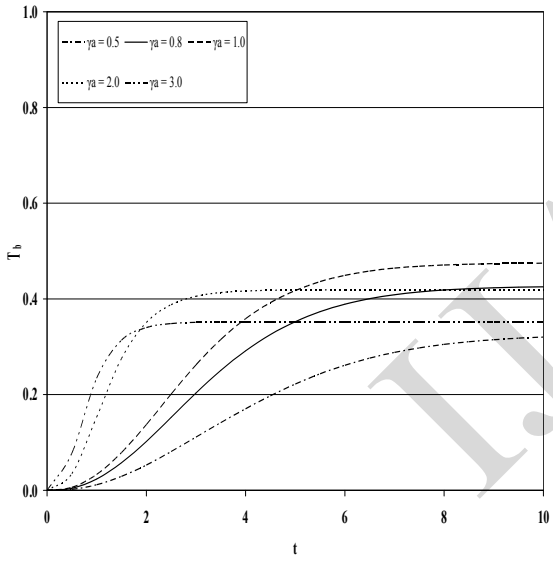


Figure 2. Mean fluid exit temperature of the 'b' fluid due to a step change of the 'a' fluid inlet temperature coupled with a step flow rate change of the 'a' fluid; initially E = R = V = NTU = 1 and γb = 1

Table 1. Mean fluid exit temperatures due to a step change of the 'a' fluid inlet temperature coupled with a step flow rate change of the 'a' fluid; initially E = R = V = NTU = 1.0; γ_b = 1.0

γ_a	Transient model steady temperatures	
	T_a	T_b
0.5	0.341	0.320
0.8	0.465	0.425
1.0	0.524	0.475
2.0	0.790	0.419
3.0	0.883	0.352

V. CONCLUSIONS

The current study directly follows from the work presented in [5, 8, 9, 10]. Conservation of energy equations are solved by using explicit finite difference techniques. The dynamic response of a heat exchanger in response to perturbations in fluid temperature and flow rates is addressed. The parametric study can be readily extended to address additional inlet conditions; for brevity, further predictions are not described in this document. The transient model can be extended to address more complicated heat transfer systems.

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ACKNOWLEDGMENT

Sincere gratitude is hereby extended to the one who never ceased in helping until this paper is structured, Dr. Karthik Silaipillayarputhur for the unwavering guidance throughout this paper.

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