Study of Biological System using Heat Transfer Equations

S. Hossain and F. A. Mohammadi
Ryerson University
Toronto, ON, Canada

Abstract—A simplified one-dimensional bioheat transfer model of the spherical living tissues in the steady state has been set up for application in heat transfer studies based on the Pennes' bioheat transfer equation and its corresponding analytical solution by using Bessel's equation has been derived in this paper. The obtained analytical solution is applied to analyze the effects of the metabolic heat generation, the tissue thermal conductivity, the blood perfusion and the coefficient of heat transfer on the temperature distribution in living tissues. A further parametric analysis has been made to reveal the tumour size, hyperactivity rate and location based on the temperature distribution profile. The results show that the derived analytic solution is useful to easily and accurately study the thermal behaviour of the biological system, and can be extended to thermal behaviour research of biological system, thermal parameter measurements, temperature field reconstruction and clinical treatment.

***Index Terms*—Bioheat transfer, Pennes' equation, analytical solution, Bessel functions, tumour**

I. INTRODUCTION

In live biological body the spatial temperature distribution in tissues plays a vital role in many physiological processes. Recent advancement in the bioheat transfer research field has paved a key foundation in hyperthermia cancer therapy, thermal diagnosis, cryogenic surgery etc. [1, 2]. The quantitative and accurate analysis of bioheat transfer is to effectively understand and model the heat transfer mechanism of the biological system.

The complexity underlying in the precise thermal analysis process of living tissues remains not only for its heterogeneity and anisotropy but also for conduction, convection, and radiation heat flow, cell's metabolism, and blood perfusion etc. Accordingly, it is very difficult to build precise and exact thermal models and most of the proposed bioheat equations [3, 4] are very complicated thus complex but not unattainable to solve analytically.
The analytical solutions of these equations have important significance in the study of bioheat transfer because they reflect actual physical feature of the equations and can be used as standards to verify the corresponding numerical results and as a proof to the reasonability of in-vitro mode analysis. Various techniques have been proposed [5-9] to obtain analytical solutions of these equations. The estimation of temperature distribution in biological tissues at steady-state in one dimensional Cartesian coordinate was presented by Zhou et al. [5]. The same analysis in cylindrical geometry with showing the parameters effect in spatial temperature distribution was depicted in [6]. Shitzer [7] presented and discussed the solution for the models of living tissue cooled at the skin surface while Shih et al. [8] solved the bioheat equation with applying sinusoidal heat flux on skin surface. For local magnetic hyperthermia, the time dependent analytical analysis of bioheat equation was performed by Gutierrez [9] in the spherical system to estimate the localized and concentrated heat required for the ablation of the cancerous cells. In this paper, the derivation of the general analytic solution for one-dimensional steady-state model of the spherical living tissue is conducted by adopting the Pennes' bioheat transfer equation [3]. The fundamental Pennes' equation and its solution are presented in the following two consecutive sections. The section 2 is devoted to present the analytic solution methodology. The application of obtained solution to the tissue's internal temperature distribution and the thermal effect of tumours are described in section 3.

II. METHODS

Pennes, in 1948, proposed a model considering cell's metabolism and blood perfusion effects on temperature distribution in live biological bodies, at steady-state condition which is written in one-dimensional spherical coordinate system as:

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right)+\frac{w_b c_b}{k}\left(T_a-T\right)+\frac{q_m}{k}=0 \quad (1)$$

where, w_b, c_b are the perfusion rate per unit volume and specific heat of blood, respectively; k is the thermal conductivity of tissue; q_m is the metabolic heat generation per unit volume; T_a and T represent the artier blood and tissue temperature, respectively.

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For axially symmetrical model the boundary conditions are described as:

$$r = 0,\ \frac{dT}{dr} = 0 \qquad (2a)$$

and

$$r = R,\ -k\frac{dT}{dr} = h_a\left(T - T_e\right) \qquad (2b)$$

where, R is the radius of the concerned tissue; h_a is the heat exchange coefficient which accounts for both the convection and radiation heat loss on the tissue surface; T_e is the ambient temperature.

To obtain dimensionless heat equation and boundary conditions the following characteristics quantities are proposed in [9, 10].

$$r^* = \frac{r}{R}, \qquad T^* = \frac{T - T_e}{T_a - T_e} \qquad (3)$$

Substituting (3) into (1) lead to:

$$\frac{1}{r^{*2}}\frac{d}{dr^*}\left(r^{*2}\frac{dT^*}{dr^*}\right) + \frac{w_b c_b R^2}{k}\left(1 - T^*\right) + \frac{q_m R^2}{k\left(T_a - T_e\right)} = 0 \qquad (4)$$

and the dimensionless boundary conditions are:

$$r^* = 0,\ \frac{dT^*}{dr^*} = 0 \qquad (5a)$$

and

$$r^* = 1,\ \frac{dT^*}{dr^*} = -\frac{h_a R}{k}T^* \qquad (5b)$$

Here, the dimensionless parameters and variables are defined as:

$$\frac{w_b c_b R^2}{k} = w^*,\ \frac{q_m R^2}{k\left(T_a - T_e\right)} = q_m{}^* \text{ and } \frac{h_a R}{k} = h_a{}^*$$

Therefore, the dimensionless governing equation is:

$$\frac{1}{r^{*2}}\frac{d}{dr^*}\left(r^{*2}\frac{dT^*}{dr^*}\right) - w_b{}^* T^* + w_b{}^* + q_m{}^* = 0 \qquad (6)$$

and the boundary conditions are:

$$r^* = 0,\ \frac{dT^*}{dr^*} = 0 \qquad (7a)$$

and

$$r^* = 1,\ \frac{dT^*}{dr^*} = -h_a{}^* T^* \qquad (7b)$$

By dropping the superscripts * and letting $w_b{}^* = \alpha$ and $w_b{}^* + q_m{}^* = \beta$, the equation (4) becomes:

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) - \alpha T + \beta = 0 \qquad (8)$$

To solve the equation (8), let's assume $T = \frac{H_{(r)}}{\sqrt{r}}$ and by differentiating we obtain,

$$\frac{dT}{dr} = r^{-1/2}H' - \frac{1}{2}r^{-3/2}H \qquad (9)$$

Substituting (9) in (8) we get,

$$\frac{1}{r^2}\frac{d}{dr}\left(r^{3/2}H' - \frac{1}{2}r^{1/2}H\right) - \alpha r^{-1/2}H = -\beta \qquad (10)$$

The homogeneous equation is:

$$\frac{1}{r^2}\frac{d}{dr}\left(r^{3/2}H' - \frac{1}{2}r^{1/2}H\right) - \alpha r^{-1/2}H = 0 \qquad (11)$$

After simplifying and multiplying by $r^{1/2}$ we get

$$r^2H'' + rH' - \left(\alpha r^2 + \left(\frac{1}{2}\right)^2\right)H = 0 \qquad (12)$$

The equation (12) is modified Bessel equation of half kind and therefore the solution is:

$$H = C_1 I_{1/2}\left(\sqrt{\alpha}r\right) + C_2 K_{1/2}\left(\sqrt{\alpha}r\right) \qquad (13)$$

According to the characteristics of Bessel's equation, we have, $C_2 = 0$. Therefore, the solution to (13) is:

$$H = C_1 I_{1/2}\left(\sqrt{\alpha}r\right) \qquad (14)$$

The exact solution in terms of $T(r)$ is:

$$T = C_1\frac{I_{1/2}\left(\sqrt{\alpha}r\right)}{\sqrt{r}} + T_p \qquad (15)$$

where, T_p is the particular solution and $T_p = \frac{\beta}{\alpha}$.

Therefore, the temperature expression considering the superscript * again:

$$T^*\left(r^*\right) = C_1\frac{I_{1/2}\left(\sqrt{\alpha}r^*\right)}{\sqrt{r^*}} + \frac{\beta}{\alpha} \qquad (16)$$

Considering the boundary condition of (7b), some simple derivations lead to:

$$\frac{dT^*}{dr^*} = C_1\left(-\frac{I_{1/2}\left(\sqrt{\alpha}r^*\right)}{2r^*\sqrt{r^*}} + \frac{\sqrt{\alpha}I_{3/2}\left(\sqrt{\alpha}r^*\right) + \frac{1}{2r^*}I_{1/2}\left(\sqrt{\alpha}r^*\right)}{\sqrt{r^*}}\right) \qquad (17)$$

$$C_1 = -\frac{h_a{}^* T^*}{\left(-0.5\ I_{1/2}\left(\sqrt{\alpha}\right) + \sqrt{\alpha}I_{3/2}\left(\sqrt{\alpha}\right) + 0.5\ I_{1/2}\left(\sqrt{\alpha}\right)\right)} \qquad (18)$$

$$T^* = -\frac{h_a{}^* T^*}{\sqrt{\alpha}I_{3/2}\left(\sqrt{\alpha}\right)}\frac{I_{1/2}\left(\sqrt{\alpha}r^*\right)}{\sqrt{r^*}} + \frac{\beta}{\alpha} \qquad (19)$$

$$T = T_e + \frac{\beta}{\alpha}\left(\frac{1}{1 + \frac{h_a{}^* I_{1/2}\left(\sqrt{\alpha}r^*\right)}{\sqrt{\alpha}r^* I_{3/2}\left(\sqrt{\alpha}r^*\right)}}\right)\left(T_a - T_e\right) \qquad (20)$$

III. RESULTS

By applying the obtained analytic solution in (20), the interior steady-state temperature of living tissues in a body organ, which could be approximated as spherical in shape (for example- female breast, buttock), in the resting person can be easily and accurately obtained, which can facilitate the further analysis of the heat

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transfer characteristics of living tissues embedding hyperactive nodule.

The effect of major thermal parameters on the temperature distribution of spherical living tissues are discussed, wherein, the typical parameter values, obtained from renowned sources [11, 12], chosen for theoretical analysis are presented in Table I.

TABLE I. Parameter values used in theoretical Analysis

Parameter	Value and Unit
w_b	3 $kg/(s.m^3)$
c_b	3850 $J/(kg.^oC)$
k	0.48 $W/(m.^oC)$
h_A	8.77 $W/(m^2.^oC)$
q_m	1085 W/m^3
T_a	37 ^oC
T_e	25 ^oC

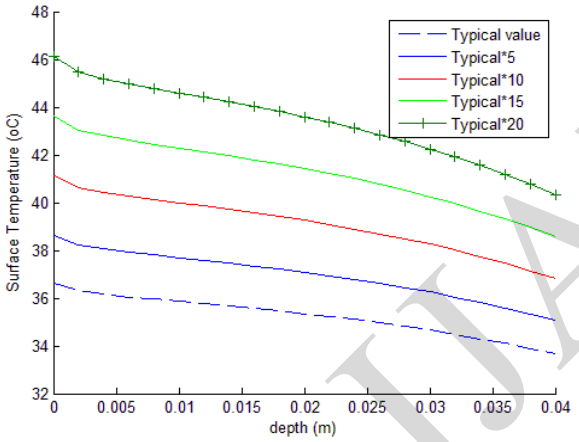


Fig. 1 Temperature Distribution for different metabolic rate

The upshot of the metabolic heat generation, the heat exchange coefficient, the blood perfusion, and the tissue thermal conductivity on temperature profile are shown in Figs. 1 to 4, respectively.

In Fig. 1, the family of graphs shows that changes in metabolic heat generation elevates the inner tissue temperature magnitudes but maintains an almost constant slope in the temperature flow path to the boundary regardless the metabolic rate.

The Fig. 2 asserts that the higher the coefficient of heat transfer, the lower the temperature near the boundary of the body. This ambient dependence phenomenon in thermal distribution of biological body produces a distinguishable elevation in skin temperature and is a tool for analyzing benign stage tumour.

The effect of blood perfusion rates on the temperature distribution is illustrated in Fig. 3. The curves indicate that the gradient of the temperature variation in redial direction decreases with increasing blood perfusion, which is a result of higher rate of heat distribution caused by the blood perfusion. Moreover, the differences between the effects of the higher blood perfusion rates on temperature distribution become smaller.

The simulation results presented in Fig. 4 shows the changes in the values of the tissue thermal conductivity has effect on body heat transfer capacity.

The effect of tumour, located at the center of the spherical model (see Fig 5) surrounded by sound tissues, on the thermal distribution of body is presented in the Fig.6. The incremental temperature distribution of a tumour of radius 5mm at depth 4 cm which has 2- 5 times higher hyperactivity than average sound tissue is presented.

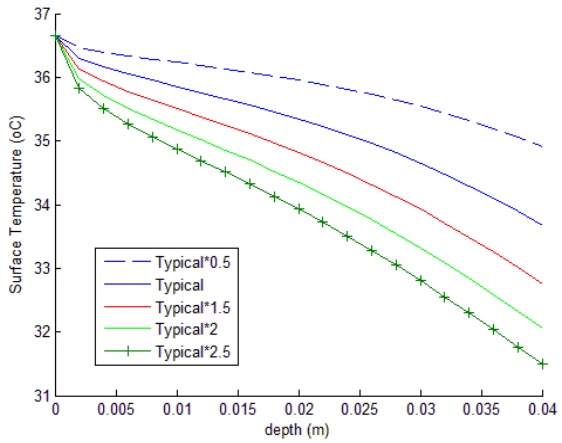


Fig. 2 Temperature Distribution for heat exchange coefficient

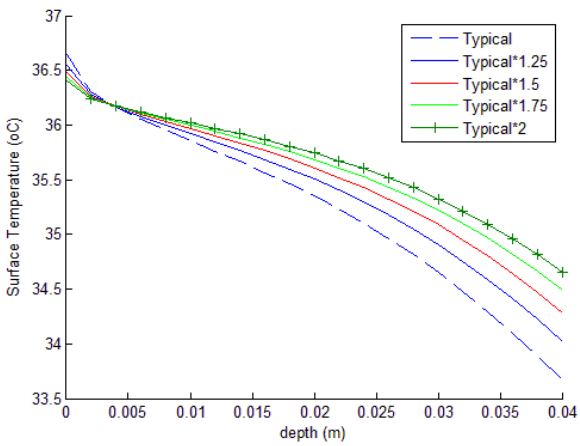


Fig. 3 Temperature Distribution for different perfusion rate

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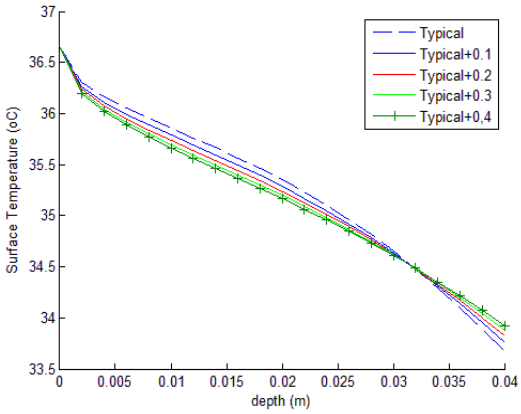


Fig. 4 Temperature Distribution for different tissue thermal conductivities

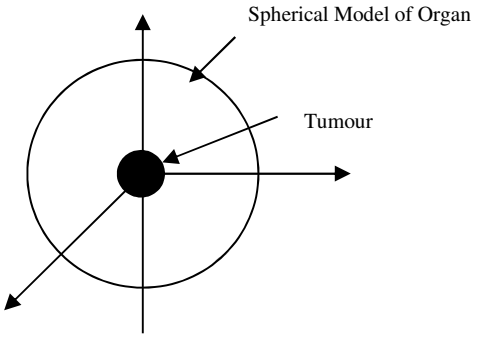


Fig. 5 Tumour at the center of the model

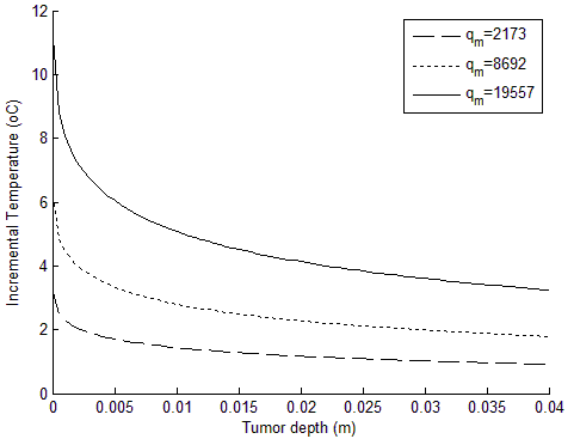


Fig. 6 Temperature Distribution for Tumour

A center tumour of different size with a constant metabolism of 5 times higher than healthy tissues is presented in Fig. 7. This figure shows that radius has no effect on the temperature established at the centre of the nodule.

The resultant thermal profile of a tumour at centered origin embedded in healthy tissues is presented in Figs. 8 and 9 for different tumour sizes and metabolism rates, respectively.

The Fig. 8 concludes that the maximum raise in the temperature at the model origin does not depend on the size of the tumour. The size has effect on the temperature magnitude at the boundary only. While the intensity influences both the temperature grow at origin and amount of temperature that reach to the boundary, it is evident from Fig 9.

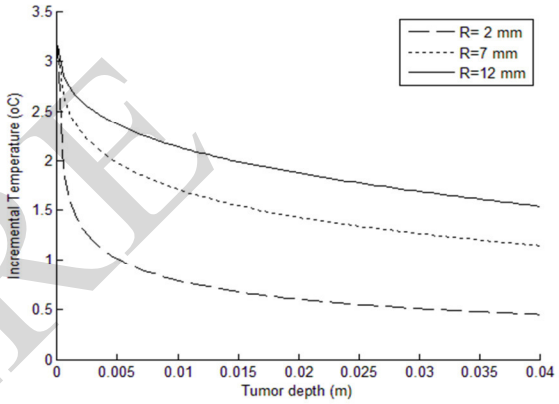


Fig. 7 Temperature Distribution of Tumour

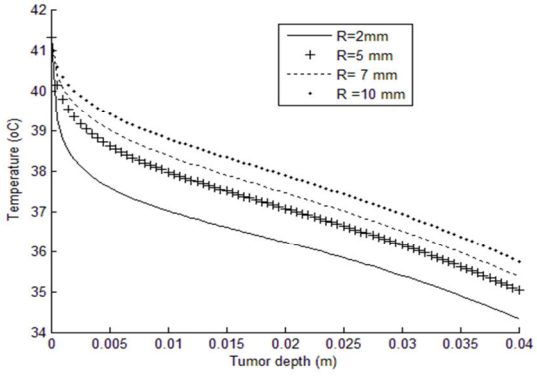


Fig. 8 Tumour in Sound Tissue

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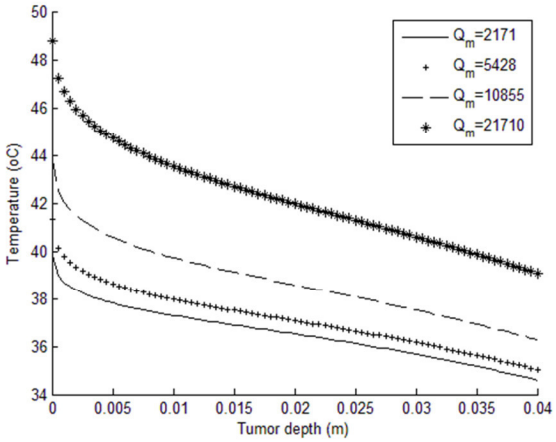


Fig. 9 Tumour in Sound Tissue

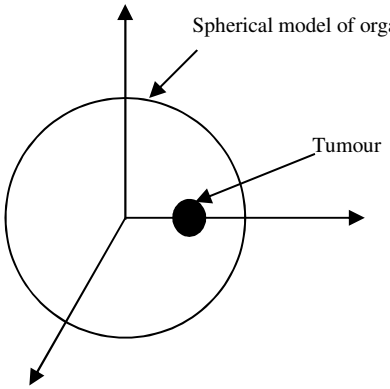


Fig. 10 Tumour not at the center of the Model

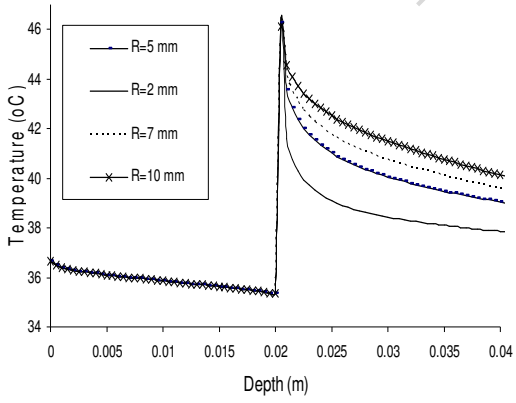


Fig. 11 Tumour in Sound Tissue

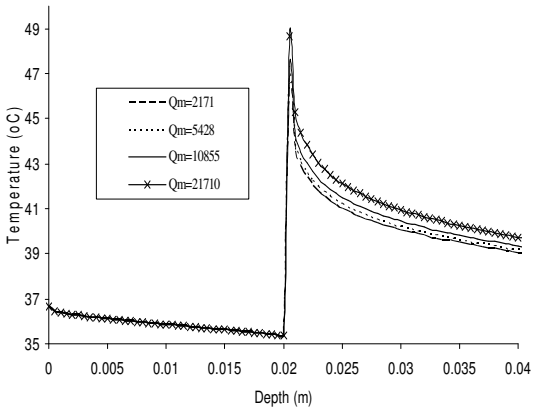


Fig. 12 Tumour in Sound Tissue

If the tumour is not located at a depth of 2 cm rather than being at the model origin, as shown in Fig. 10, the temperature flow in the outward direction for different tumour sizes and metabolism rates are presented in Figs. 11 and 12, respectively.

It can be seen from Fig. 11 that an abrupt change in temperature profile at the tumour position makes a constant vertex at 46°C for all the values of radius. The analysis also shows that the increasing hyperactivity rate of tumour tissues increases both the temperature magnitude at the tumour location and at boundary which is conveyed by Fig. 12.

IV. CONCLUSIONS

In this paper, the steady-state bioheat transfer model of the spherical living tissues has been analyzed based on Pennes' equation, and the corresponding equation has been solved in one-dimensional domain. The Bessel's equation involving analytical solution is obtained in this study is useful for describing the tissue temperature distribution in radial direction.

Taking account of the parametric studies, the thermal conductivity and blood perfusion effects are neglected for further analysis in case of tumours, though the thermal conductivity of the tumour cells is somewhat increase from that of sound tissue but it plays an insignificant effect on temperature profile and likewise the blood perfusion rate in a particular organ is assumed to be undisturbed by the cancerous cells.

Observing the linearity in thermal profile with the variations in tissue metabolism, the superposition theorem is employed to compute the temperature distribution of spherical shaped organ embedding tumour within. Analysis with varying tumour location is performed using the concept of coordinate origin shifting.

The temperature reach to the skin boundary bear the signature of availability of tumour in a particular organ

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of biological body and analyzing and mapping the temperature reveal some idea about the clinical features of tumour. This aspect needs further analysis to pinpoint the correlation among the tumour parameters with temperature at skin surface so that an invasive, non-contact, radiation free diagnostic tool can be invoked employing infrared (IR) thermogram.

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